MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XC. DECEMBER 13, 1929. NO. 2

Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Joseph Bevan Braithwaite, Lea Wood, Woodside Avenue, Muswell Hill, London, N. 10 (proposed by W. Goodacre) ;

Walter Frederick Kibble, M.A., Professor of Mathematics, Free Christian College, Madras, S. India (proposed by F. J. M. Stratton) ;

Emlyn Samuel Phillips, 40 Barrington Road, London, N. 8 (proposed by W. J. Coombs) ; and

John Henry Wills, B.Sc., Bethany House School, Goudhurst, and Trevelyan, Bodmin, Cornwall (proposed by H. W. Unthank).

Eighty-two presents were announced as having been received since the last meeting, including amongst others :—

Naples, Osservatorio di Capodimonte, Astrographic Catalogue (Catania zones) vol. iv. pts. 3 and 4.

Ludwig Darmstaedter, Handbuch zur Geschichte der Naturwissenschaften und der Technik (presented by Dr. Knobel).

Harold Jeffreys, The Future of the Earth.

Johannes Kepler, Neue Astronomie (presented by Dr. Comrie).

E. A. Milne, The Aims of Mathematical Physics.

G. W. Ritchey, The Development of Astro-photography and the

Great Telescopes of the Future (presented by MM. Saint Gobain, Chaunis, and Cirey).

Southampton Harbour Board, Official Tide Tables for Southampton, 1930.

Mr. T. Whitwell and Mr. W. M. Witchell were appointed auditors of the Treasurer's accounts for 1929.

On the Central Intensities of some Absorption Lines in the Solar Spectrum. By R. v. d. R. Woolley, M.Sc., B.A., Gonville and Caius College, Cambridge.

§ 1. *Introduction and Summary.* — Theoretical formulæ for intensities inside Fraunhofer lines have been given by Schuster (1), Milne (2), and Eddington (3), and observations have been published by Unsöld (4) and Minnaert (5). It is usual in recent practice to disregard the central intensity of the line and to fit the observations to the theoretical contour at a point where the intensity inside the line has fallen to one-half that of the continuous background just outside the line.

In the three model atmospheres mentioned above there are respectively one, two, and three kinds of absorption. Schuster's pioneer paper considers only one kind of absorption and the fit with the observed contour determines the number of atoms above 1 sq. cm. of the photosphere. Milne * takes into account collisions between the atoms and shows (6) that these appear as "true" absorption. Eddington further takes into account ordinary continuous absorption. There would be no point in distinguishing between apparent absorption, due to super-elastic collisions, and ordinary absorption if they did not affect the radiation in different ways. Eddington shows that this is so, the difference being that apparent or collision absorption due to a decrease of emission is equally distributed in all directions, while of course the ordinary absorption is not the same in all directions, there being a net outflow of radiation.†

We shall in § 2 use a formula given by H. A. Wilson (7) and Unsöld (4) and apply it to the centre of the H and K lines of Ca^+; only certain lines are considered in order to avoid complications due to interlocking (8). In § 3 the Doppler effect is taken into account. Applying these results derived from purely physical principles to the formulæ of Schuster (§ 4) and Milne (§ 5), we find that the central blackening given is far too great. In § 6 we investigate the effect of a turbulent motion suggested by Milne (9) and supported by some observations of

* The writer learns from Professor Milne that he takes the effect of collisions to be the same inside the line as outside it, thus, in effect, omitting them. His κ is the coefficient of general absorption.

† There are also differences in the corresponding emission processes. See Eddington (*loc. cit.*, p. 623) or the footnote to § 7.

Minnaert (10). This modifies the results previously obtained but does not increase the intensities sufficiently to agree with observation.

In § 7 we consider Eddington's model of the solar atmosphere and determine from the breadth of the line the continuous absorption, and from the central intensity the collision absorption. The observations then give the number of collisions per second, and hence the mean pressure of the layers concerned. The pressures so found are of the right order of magnitude as compared with values given by Milne (11) from other considerations. In § 8 we consider the effect of the chromosphere, as an upper layer in which collisions are very rare. In § 9 we consider the principal lines of Ba^+ (4554 and 4934). These lines are similar to the H and K lines of Ca^+, but are very much narrower in the Sun's spectrum and there is no Ba^+ high in the chromosphere.

§ 2. Let s_ν be the coefficient of line absorption : that is, if a monochromatic beam of frequency ν traverses unit length in the atmosphere, which has a density ρ, then the fraction ρs_ν of the incident energy is absorbed (and, of course, re-emitted. The process is called scattering by some writers). We take as our fundamental formula the form

$$s_\nu = \frac{s_0}{1 + a^2(\nu - \nu_0)^2} . \quad . \quad . \quad . \quad (2.1)$$

given by H. A. Wilson (7) where $s_0 (= s_{\nu 0})$ and a are constants for any particular absorption line. Unsöld (4) uses

$$s_\nu = \frac{1}{M} \frac{2\pi e^4}{3m^2c^4} \frac{\nu_0^2}{(\nu - \nu_0)^2} f \quad . \quad . \quad . \quad (2.2)$$

where M is the mass of the atom concerned, f is the " oscillator strength," e and m the charge and mass of the electron. Unsöld's formula breaks down near the central frequency, but he states that the total amount of energy scattered per second by an atom from radiation of density ρ_ν is, on both classical and quantum theories,

$$\frac{\pi e^2}{m} \rho_\nu \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

We can now determine s_0 and a in Wilson's formula, since, by comparison with Unsöld's formula, when $\nu - \nu_0$ is large we have

$$\frac{s_0}{a^2} = \frac{1}{M} \frac{2\pi e^4}{3m^2c^4} \nu_0^2 f \quad . \quad . \quad . \quad . \quad (2.41)$$

Further, the total scattering is

$$\int_{\nu=0}^{\infty} \frac{M s_0 d\nu}{1 + a^2(\nu - \nu_0)^2} = M\pi \frac{s_0}{a}.$$

Hence from (2.3)

$$\frac{M\pi s_0}{a} = \frac{1}{c} \frac{\pi e^2}{m} f \quad . \quad . \quad . \quad . \quad (2.42)$$

since $\Sigma f = 1$ for the lines corresponding to a given state of the atom.

Inserting numerical values, we find for

$$\left.\begin{aligned} H_1(f = \tfrac{1}{3})_1\, s_0 &= 3{\cdot}77\ 10^{12};\ a = 8{\cdot}84\ 10^{-8} \\ K_1(f = \tfrac{2}{3})_1\, s_0 &= 7{\cdot}41\ 10^{12};\ a = 8{\cdot}68\ 10^{-8} \end{aligned}\right\} \quad . \quad . \quad (2.43)$$

§ 3. *Doppler Effect.*—We consider an assembly of N atoms which have a Maxwellian distribution in velocity corresponding to a temperature T. Then the number of atoms whose velocity lies between u and $u + du$ in directions making angles between θ and $\theta + d\theta$ with the radius vector is

$$\frac{2N}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{\frac{3}{2}} u^2 e^{-\frac{Mu^2}{2RT}} du \tfrac{1}{2} \sin\theta d\theta \quad . \quad . \quad . \quad (3.1)$$

Now consider a beam of light of frequency τ travelling along the radius vector. An atom receives the light as having a frequency τ' where

$$\tau' = \tau\left(1 - \frac{u}{c}\cos\theta\right) \quad . \quad . \quad . \quad (3.11)$$

and the amount of energy scattered per second by this atom is

$$\frac{Ms_0}{1 + a^2(\nu' - \nu_0)^2}\rho_\nu \quad . \quad . \quad . \quad (3.12)$$

where

$$\frac{\nu'}{\nu} = \frac{\tau}{\tau'} = 1 + \frac{u}{c}\cos\theta \quad . \quad . \quad . \quad (3.13)$$

and ρ_ν is the energy density of the radiation.

Hence the total amount scattered per second by the assembly is

$$MN s_0 \rho_\nu \frac{2}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} \int_{u=0}^{\infty} du \int_{\theta=0}^{\pi} \frac{u^2 e^{-\frac{Mu^2}{2RT}} \tfrac{1}{2} \sin\theta d\theta}{1 + a^2\left\{\nu\left(1 + \frac{u}{c}\cos\theta\right) - \nu_0\right\}^2} \quad (3.2)$$

That is, the assembly behaves as if each atom had a scattering coefficient s_ν', where

$$\begin{aligned} s_\nu' &= s_0 \frac{2}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} \int_{u=0}^{\infty} du \int_{\theta=0}^{\pi} \frac{\tfrac{1}{2} u^2 e^{-\frac{Mu^2}{2RT}} \sin\theta d\theta}{1 + a^2\left\{\nu\left(1 + \frac{u}{c}\cos\theta\right) - \nu_0\right\}^2} \\ &= s_0 \frac{1}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{\frac{3}{2}} \frac{c}{a\nu} \int_0^{\infty} u e^{-\frac{Mu^2}{2RT}} \left[\tan^{-1} a\left(\nu - \nu_0 + \frac{\nu u}{c}\right)\right. \\ &\qquad \left. - \tan^{-1} a\left(\nu - \nu_0 - \frac{\nu u}{c}\right)\right] du \end{aligned} \quad (3.3)$$

It is unnecessary to attack the integral in (3.3) directly, as we can make some approximations.

If we consider frequencies such that $\left(\frac{\nu-\nu_0}{\nu}\right)^2$ is large compared with

$$\frac{3\overline{u^2}}{c^2} = \frac{3RT}{Mc^2}$$

and that

$$a\left(\nu - \nu_0 - \frac{\nu\bar{u}}{c}\right)$$

is so large that we may put

$$\tan^{-1} a\left(\nu - \nu_0 - \frac{\nu u}{c}\right) = \frac{\pi}{2} - \frac{1}{a\left(\nu - \nu_0 - \frac{\nu u}{c}\right)},$$

then the integrand * becomes

$$ue^{-\frac{Mu^2}{2RT}}\left\{\frac{\pi}{2} - \frac{1/a}{\nu - \nu_0 + \frac{\nu u}{c}} - \frac{\pi}{2} + \frac{1/a}{\nu - \nu_0 - \frac{\nu u}{c}}\right\}$$

$$= ue^{-\frac{Mu^2}{2RT}} \frac{2\nu u}{ac(\nu-\nu_0)^2}\left\{1 + \left(\frac{\nu u}{c(\nu-\nu_0)}\right)^2\right\},$$

neglecting higher powers of

$$\frac{\nu}{\nu-\nu_0} \cdot \frac{u}{c}$$

(3.3) becomes

$$s_\nu' = s_0 \frac{1}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} \frac{c}{a\nu}\left[\frac{2\nu}{ac(\nu-\nu_0)^2} \frac{\frac{1}{2}\sqrt{\pi}}{\left(\frac{M}{2RT}\right)^{3/2}} + \frac{2\nu^3}{ac^3(\nu-\nu_0)^4} \frac{\frac{3}{4}\sqrt{\pi}}{\left(\frac{M}{2RT}\right)^{5/2}}\right].$$

Neglecting the second term in the bracket we get

$$s_\nu' = \frac{s_0}{a^2(\nu-\nu_0)^2} \quad . \quad . \quad . \quad . \quad (3.31)$$

which is (2.2). Thus the Doppler effect only affects s_ν for small values of $\nu - \nu_0$, as we should expect. The approximations we have used hold at least down to $\frac{\nu-\nu_0}{\nu} = 10^{-4}$ for Ca^+ in the Sun. As the "half-width" is at $\frac{\nu-\nu_0}{\nu} = 1{\cdot}6\ 10^{-3}$, the Doppler effect only affects the extreme centre. Putting $\nu = \nu_0$ in (3.3) we get $\tan^{-1}\frac{a\nu u}{c} = \frac{\pi}{2}$ over

* This approximation is of course not valid over the whole range of the integral, from $u=0$ to $u=\infty$. But we can neglect the range $u > u_1$ (say) which will not contribute much to the integral, since there is the term $e^{-\frac{Mu^2}{2RT}}$ in the integrand.

the effective range of the integral, since $\frac{\bar{u}}{c} = 10^{-5}$ and $av = 10^7$, (3.3) becomes

$$s_0' = s_0 \frac{1}{\sqrt{\pi}} \left(\frac{M}{2RT}\right)^{3/2} \frac{c}{av_0} \pi \int_{u=0}^{\infty} u e^{-\frac{Mu^2}{2RT}} du$$

$$= s_0 \frac{c}{2av_0} \left(\frac{\pi}{2} \frac{M}{RT}\right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (3.32)$$

We have another approximation that $s_\nu' = \frac{1}{2} s_0'$, where

$$\left| \frac{\nu' - \nu_0}{\nu_0} \right| = \frac{\bar{u}}{c} \sim 10^{-5},$$

since for this value of ν the second term in the bracket contributes approximately nothing to the integral : the first term contributes

$$\frac{\pi}{2} \int_0^{\infty} u e^{-\frac{Mu^2}{2RT}} du$$

as before.

Inserting numerical values into (3.32) with T = 6000°, we get

$$s_0' = s \times 2{\cdot}55 \, . \, 10^{-3}.$$

Thus for

H, $s_0' = 8{\cdot}6 \, 10^9$

and for K, $s_0' = 1{\cdot}7 \, 10^{10}$ . . . . . (3.33)

§ 4. *Central Intensity predicted by Schuster's Formula.*—Let $r^*(\nu)$ denote the ratio of the intensity of radiation of frequency ν inside the line to the intensity of the background just outside the line. Schuster deals with N atoms placed in front of a radiating surface, having a scattering coefficient s_ν but no general absorption. If M is the mass of each atom, then

$$r^*(\nu) = \frac{1}{1 + N.M.s_\nu} \quad . \quad . \quad . \quad . \quad (4.1)$$

From the table published by Minnaert (*loc. cit.*) we find for the K line

$$r^*(\nu_1) = \tfrac{1}{2} \quad \text{where} \quad \nu_1 - \nu_0 = 1{\cdot}02 \, 10^{12}.$$

Using the results (2.43) we find s_{ν_1}, which gives $N = 1{\cdot}57 \, 10^{19}$. With this value of N we find, using the correction for Doppler effect † (3.33),

$$r^*(\nu_0) = 5{\cdot}6 \, 10^{-8}.$$

It is of interest to calculate $r^*(\nu_2)$ where $|\nu_2 - \nu_0| = 10^{10}$, so that $|\nu_2 - \nu_0|$ is within a resolving power of 10^5, *i.e.*

$$\frac{\nu_2 - \nu_0}{\nu_0} > 10^{-5}.$$

† We found in § 3 that this correction is only important near the centre : we use (2.43) at $r = \frac{1}{2}$. Without the Doppler effect, the value of $r^*(\nu_0)$ would be $1{\cdot}3 \, 10^{-10}$.

We have noted that here approximately $s_{\nu_2}' = \frac{1}{2}s_0'$, so that

$$r^*(\nu_2) \doteqdot 10^{-7}.$$

Thus an instrument with this resolving power should find a central intensity of order 10^{-7}.

The effect of introducing the Doppler effect is shown in the following table for the K line :—

$\nu - \nu_0$.		10^{12}.	10^{11}.	10^{10}.	10^{9}.	0.
$r^*(\nu)$	Uncorrected	0·5	$5\ 10^{-3}$	$5\ 10^{-5}$	$5\ 10^{-7}$	$1{\cdot}3\ 10^{-10}$
$r^*(\nu)$	Corrected for Doppler Effect.	0·5	$5\ 10^{-3}$	10^{-5}	10^{-7}	$5{\cdot}6\ 10^{-8}$

From this table we see that the Doppler effect only modifies a small section of the contour near the centre of the line, the centre being filled in at the expense of its immediate neighbourhood. The total amount of energy scattered per sec. by an atom on top of the atmosphere is not much affected by this modification. To see this, let I be the intensity of the continuous background. Then the energy scattered per sec. between frequencies ν_0 and ν by an atom on top of the atmosphere is

$$\int_{\nu_0}^{\nu} \mathrm{I}Ms_\nu r^*(\nu)d\nu \quad . \quad . \quad . \quad . \quad (4.2)$$

If we take no account of the Doppler effect

$$r^*(\nu) = \frac{\mathrm{I}}{1 + \mathrm{N}Ms(\nu)},$$

and (4.2) becomes

$$\frac{\mathrm{I}\sigma_0/a}{\sqrt{1 + \mathrm{N}\sigma_0}} \tan^{-1} \frac{a(\nu_1 - \nu_0)}{\sqrt{1 + \mathrm{N}\sigma_0}} \quad . \quad . \quad . \quad (4.21)$$

where $\sigma_\nu = Ms_\nu$.

Since $\dfrac{a}{\sqrt{1 + \mathrm{N}\sigma_0}} = 3{\cdot}10^{-12}$ for K, less than one-third is contributed by the range in which the Doppler effect produces any modification ($|\nu - \nu_0| < 10^{11}$).

If we take as an approximate contour

$$r^*_\nu = \frac{\mathrm{I}}{1 + \mathrm{N}\sigma_\nu} \qquad |\nu - \nu_0| \geqslant 5{\cdot}10^{10}$$

$$r^*_\nu = \text{const} = 1{\cdot}10^{-6} \qquad |\nu - \nu_0| < 5{\cdot}10^{10},$$

the two sections contribute respectively approximately $\mathrm{I}\sigma_0/a \cdot \dfrac{\pi}{\sqrt{1 + \mathrm{N}\sigma_0}}$ and $\mathrm{I}\sigma_0/a\pi(2r^*(\nu_0))$, the contribution from the outer section being about 37 times that from the inner since $\dfrac{\mathrm{I}}{\sqrt{1 + \mathrm{N}\sigma_0}} = 3{\cdot}7\ 10^{-5}$. Hence we can calculate the amount scattered per sec. by the atom from the simple formula

$$2\int_{\nu=\nu_0}^{\infty} \mathrm{I}\frac{\sigma_\nu}{1 + \mathrm{N}\sigma_\nu}d\nu = \frac{\pi \mathrm{I}\sigma_0/a}{\sqrt{1 + \mathrm{N}\sigma_\nu}} \quad . \quad . \quad . \quad (4.22)$$

without serious error. If we take I as the black-body intensity B corresponding to temperature T = 6000°, we shall find the momentum absorbed per sec. by the atom from the scattering. The momentum acquired per sec. by unit mass is

$$\frac{1}{c}\pi B\frac{s_0}{a}\frac{1}{\sqrt{1+N\sigma_0}} \quad . \quad . \quad . \quad . \quad (4.23)$$

Inserting numerical values, this comes to about 5 cms./sec.^{2}. This is the amount absorbed in the K line. Slightly less will be absorbed from the H line, making a total amount of about 8·5 cms./sec.^{2}, so that an atom on top of Schuster's atmosphere cannot support itself by selective radiation-pressure against $g = 2{\cdot}74\ 10^4$ cms./sec.^{2}.

§ 5. *Central Intensity predicted by Milne's Formula.*—Milne gives

$$r(\nu) = \frac{2\xi_\nu}{1+\xi_\nu}\cdot\frac{a_\nu + b_\nu\xi_\nu}{a_\nu + b_\nu} \quad . \quad . \quad . \quad (5.1)$$

where

$$\xi_\nu^2 = \frac{\kappa}{\kappa + s_\nu}\dagger \text{ and } \frac{b_\nu}{a_\nu} = \frac{1}{4}\frac{h\nu_0}{RT}\cdot\frac{1}{1-e^{-\frac{h\nu}{RT}}} = 0{\cdot}8$$

for H and K in the Sun.

Using the same observation of Minnaert, viz. that $r(\nu_1) = \frac{1}{2}$, where $\nu_1 - \nu_0 = 1{\cdot}02.10^{12}$, we find that $\kappa = 282$. (The disagreement with Milne's value 5·5 § is only apparent, because here the atmosphere is supposed to consist of calcium only. It is this value of κ which must be used in connection with the values (2.43), (3.33) for s_0 and a.)

We can at once calculate

$$r(\nu_0) = 1{\cdot}44\ 10^{-4}$$

and

$$r(\nu_2) = 2{\cdot}34\ 10^{-4},$$

so that an instrument of resolving power 10^5 should show a central intensity of order $2{\cdot}10^{-4}$. Though considerably better than the Schuster formula, Milne's contour predicts far too small a central intensity, the observed central intensity being 7 per cent. (Minnaert, *loc. cit.*).

The following table gives $r(\nu)$ for a set of values of $(\nu - \nu_0)$ in comparison with the value of $r(\nu)$ when there is no Doppler effect :—

$\nu-\nu_0$.		10^{12}.	10^{11}.	10^{10}.	10^9.	0.
$r(\nu)$	Uncorrected	0·5	0·047	$5{\cdot}10^{-3}$	$5{\cdot}10^{-4}$	$5{\cdot}5\ 10^{-6}$
$r(\nu)$	Corrected for Doppler Effect.	0·5	0·047	$2{\cdot}4\ 10^{-4}$	$1{\cdot}44\ 10^{-4}$	$1{\cdot}44\ 10^{-4}$

The momentum absorbed by an atom above a Milne atmosphere is chiefly supplied by the small range which is affected by the Doppler

† κ is the coefficient of "true" absorption. The corresponding emission is κB_ν per unit mass. $\kappa/s_\nu \propto \varrho$ approximately. The physical process responsible for this absorption (approximately independent of wave-length) is superelastic collision between the atoms.

§ See Milne's revision of this value (*Phil. Trans.*, A, **228**, 421, 1929).—EDITORS.

effect. This is exactly the reverse of the case discussed in § 4. The amount of momentum acquired per sec. from selective radiation-pressure is again too small to support the atom against solar gravity.

For convenience, we take as an approximation to (5.1)

$$r(\nu) = \sqrt{\frac{\kappa}{\kappa + s_\nu}} = \sqrt{\frac{1 + x^2}{1 + s_0/\kappa + x^2}} \quad . \quad . \quad (5.11)$$

where $x = a(\nu - \nu_0)$.

Then the energy scattered per sec. by an atom between frequencies ν and ν_0 is

$$\int_{\nu_0}^{\nu} I\sigma_\nu r(\nu) d\nu = I\frac{\sigma_0}{a}\int_0^{a(\nu-\nu_0)} \frac{dx}{\sqrt{1 + x^2}\sqrt{s_0/\kappa + x^2}} \quad . \quad (5.2)$$

Since $\frac{s_0}{\kappa}$ is large compared with unity we can split the integral into two parts if $a(\nu - \nu_0)$ is large.

Case I.

$$a(\nu - \nu_0) \text{ small compared with } \frac{s_0}{\kappa} = b, \text{ say,}$$

(5.2) becomes

$$I\frac{\sigma_0}{a}\int_0^b \sqrt{\frac{\kappa}{s_0}} \frac{dx}{\sqrt{1 + x^2}} = I\frac{\sigma_0}{a}\sqrt{\frac{\kappa}{s_0}} \sinh^{-1} b \quad . \quad . \quad (5.21)$$

Case II.

$$a(\nu - \nu_0) \text{ of order } \frac{s_0}{\kappa} = c^2, \text{ say,}$$

(5.2) becomes

$$I\frac{\sigma_0}{a}\left\{\int_0^c \sqrt{\frac{\kappa}{s_0}} \frac{dx}{\sqrt{1 + x^2}} + \int_c^{c^2} \frac{dx}{x\sqrt{\frac{s_0}{\kappa} + x^2}}\right\} \quad . \quad . \quad (5.22)$$

In particular, for the total amount of energy scattered in all frequencies per sec.,

$$2I\int_{\nu_0}^{\infty} \sigma_\nu \sqrt{\frac{\kappa}{\kappa + s_\nu}} d\nu = 2I\frac{\sigma_0}{a}\left\{\int_0^d \sqrt{\frac{\kappa}{s_0}} \frac{dx}{\sqrt{1 + x^2}} + \int_d^{\infty} \frac{dx}{x\sqrt{\frac{s_0}{\kappa} + x^2}}\right\}$$

$$\left(\text{where } d^4 = \frac{s_0}{\kappa}\right)$$

$$= 4I\frac{\sigma_0}{a}\sqrt{\frac{\kappa}{s_0}} \log_e 2\left(\frac{s_0}{\kappa}\right)^{1/4} \quad . \quad . \quad . \quad (5.23)$$

This is the amount scattered from the unmodified contour.

Since

$$\frac{d}{a} = \frac{1}{a} \cdot (s_0/\kappa)^{1/4} = 4{\cdot}6\ 10^9,$$

more than half comes from the range $|\ \nu - \nu_0\ | < 10^{10}$.

From the table, we can take as a rough approximation to the modified contour

$$r(\nu) = \text{const.} = 1{\cdot}5 \,.\, 10^{-4} \text{ inside } | \nu - \nu_0 | < 10^{10}.$$

The amount of energy scattered from this range is then

$$2I\frac{\sigma_0}{a} r(\nu) \tan^{-1} a(\nu - \nu_0) = \pi I \frac{\sigma_0}{a} r(\nu) \quad . \quad . \quad (5.3)$$

Inserting numerical values,

$$\begin{aligned} (5.3) &\text{ becomes } I\sigma_0 \times 5{\cdot}3\ 10^{-4}, \\ (5.23) &\quad ,, \quad I\sigma_0 \times 1{\cdot}6\ 10^{-4}, \end{aligned}$$

so that the Doppler modification increases the amount of momentum that can be picked up by an atom above the atmosphere. It is not necessary to consider the Doppler effect in the absorbing atom, since very few of the atoms will have their central absorbing frequencies shifted off the straight part of the contour

$$| \nu - \nu_0 | < 10^{10}.$$

The momentum acquired per sec. by unit mass is approximately

$$\frac{I}{c}\pi B \frac{s_0}{a} r(\nu_0) = 22{\cdot}5 \text{ cms./sec.}^{-2}$$

from the K line. As before, the H line contributes slightly less momentum and the atom is not supported against $g = 2{\cdot}74\ 10^4$ cms./sec.$^{-2}$.

§ 6. *Radial Ejection of* Ca^+ *Atoms.*—Before passing on to consider Eddington's model of the atmosphere, we will consider a hypothesis due to Milne (10), who suggests that Ca^+ atoms are ejected from lower layers into the chromosphere. They have to cover an average distance of 5000 km. in $1{\cdot}9\ 10^2$ secs. If this is so, we should expect the H and K lines to show a Doppler shift. Minnaert (11) found an " asymmetry " of the K line which can be interpreted as a shift of about $\frac{1}{4}$ angstrom = 20 km./sec. This is in excellent agreement with the value suggested by the theory. It may be urged as an objection to this hypothesis that the H and K lines are used for measuring radial velocities of stars and exhibit no anomalies as compared with other lines ; but we only suppose this ejection to occur in stars possessing a calcium chromosphere, and broad H and K lines, whose exact position cannot be determined without spectro-photometry, and would not be used for radial velocity determinations.

Ca^+ atoms are ionised in the outer chromosphere to Ca^{++}, and in this condition they fall back into the photosphere where they capture an electron and are ejected by radiation-pressure as Ca^+. The velocities of ejection will be distributed over a certain range, dependent on the depth into the photosphere to which the Ca^{++} atom has fallen. The effect of this distribution in velocity is similar to the temperature Doppler effect considered in § 3, except that we are not sure that the distribution in velocity is Maxwellian. (It will show a tendency to be

confined to the vertical direction.) We can get some idea of the order of magnitude of the effect to be expected if we suppose that the distribution is Maxwellian with a most probable velocity of 25 km./sec. This corresponds to a temperature of about 400,000°, and from (3.32) we get

$$s_0'' = s_0 \times 3{\cdot}06\ 10^{-4} \quad . \quad . \quad . \quad (6.1)$$

We get

$$r^*(\nu_0) = 6{\cdot}10^{-5}$$

and

$$r(\nu_0) = 3{\cdot}4\ 10^{-4}.$$

§ 7. We have now seen that both the Schuster and the Milne formulæ break down completely at the centre of the line, and we now investigate the behaviour of Eddington's model atmosphere (3).

The notation employed is

l_ν, Coefficient of line absorption = Milne's s_ν
k, ,, ,, continuous absorption = Milne's κ_ν
ϵl_ν, ,, ,, absorption through superelastic collisions.†
(Does not occur in Milne's work.)

Under certain simplifying conditions, the contour is given by Eddington's equation (17),

$$r(\nu) = \frac{1 + \frac{2}{3}q}{1 + \eta + \frac{2}{3}q} \quad . \quad . \quad . \quad . \quad (7.1)$$

where

$$\eta = \frac{l_\nu}{k} \quad \text{and} \quad q^2 = 3(1 + \eta)(1 + \epsilon\eta).$$

We suppose that ϵ is small, and that a fit is made with the observed "half width" $\Delta\nu$ for which $r = \frac{1}{2}$. This determines the value of k from the known value of $l(\nu_0 + \Delta\nu)$. On the other hand, at the centre we have η very large compared with unity, and we get

$$q^2 \sim 3\epsilon\eta^2.$$

(7.1) becomes

$$r(\nu_0) = \frac{\sqrt{3\epsilon}}{3/2 + \sqrt{3\epsilon}} = \tfrac{2}{3}\sqrt{3\epsilon} \quad . \quad . \quad . \quad (7.11)$$

which does not depend on $l_{\nu 0}$. The central intensity depends on the effect of collisions, and we avoid the intense central blackening given by the other contours: we can make a determination of ϵ from

† The coefficient of absorption is the fraction of the incident energy absorbed when radiation traverses unit mass of the atmosphere. Eddington distinguishes three different coefficients of absorption, corresponding to different atomic processes, because they affect the radiation in different ways. (Eddington's eqn. (4), *M.N.*, 89, 623, exhibits the precise differences; the absorption due to collisions (the term ϵl_ν) is independent of the direction of flow of the radiation considered: the re-emission from l_ν (scattering) is proportional to the actual radiation J_ν, while the re-emission from the other two terms is proportional to the equilibrium value J_T.)

the observed central intensity. Before making a numerical estimate we pass on to Eddington's more exact formula (22) †

$$r(\nu) = \frac{b + aq}{b + a\sqrt{3}} \cdot \frac{1 + \frac{2}{3}\sqrt{3}}{1 + \eta + \frac{2}{3}q} \quad . \quad . \quad . \quad (7.2)$$

where a and b are constants. Proceeding as before we find

$$r(\nu_0) = \frac{1 + \frac{2}{3}\sqrt{3}}{b/a + \sqrt{3}}\sqrt{(3\epsilon)} \quad . \quad . \quad . \quad (7.21)$$

For the Ca^+ lines $b/a = 2{\cdot}25$, by Eddington (26).

Taking for the half width $\Delta\nu = 1{\cdot}02 \cdot 10^{12}$ sec.$^{-1}$, from (7.2) we get $k = 338$. (7.21) gives $\epsilon = 1{\cdot}43\ (r(\nu_0))^2$. Thus, if $r(\nu_0) = 0{\cdot}07$ (as given by Minnaert), $\epsilon = 7 \cdot 10^{-3}$.

Assuming that the average life of an excited Ca^+ atom is $r(\nu_0) \times 1{\cdot}6 \cdot 10^{-7}$ sec. (12), we can calculate the pressure corresponding to this value of ϵ from the mean free path. If the average life of an excited atom is t, we know from the average temperature velocity the average excited path l. An excited atom has a probability ϵ of being struck while in this condition: hence the mean free path $\lambda = l/\epsilon$. We have further the relation between pressure and mean free path (13),

$$p = \frac{1}{\lambda}\frac{144}{\pi}\frac{(\mathrm{RT})^3}{e^4} \quad . \quad . \quad . \quad . \quad (7.3)$$

where R is Boltzmann's constant, and e is the charge on the electron.

At a temperature T = 6000° the mean temperature velocity is 1·77 km./sec. We find the following table of values of p corresponding to different values of $r(\nu_0)$:—

$r(\nu_0)$.	p dynes per sq. cm.
0·1	$24{\cdot}8\ 10^2$
0·07	7·0
0·03	3·8
0·01	2·5

Compare this with Milne's value for $p = 4{\cdot}65\ 10^2$ dynes per sq. cm. at a depth $\tau = 1/3$ in the Sun (14).

Notice that the Doppler effect does not now come into consideration and that it is unnecessary to consider the finite resolving power of the instrument, since the intensity is independent of l_ν where l_ν is large.

These pressures represent means for the whole layers responsible for producing the absorption lines. The question of the legitimacy of including the chromosphere is discussed in the next section.

§ 8. We have now to consider the following objection to the treatment in the preceding section. We have attributed the high central intensity to collisions: but the radiation has to pass through the chromosphere, where the pressure is so low that collisions are extremely

† *M.N.R.A.S.*, **89**, 625, 1929.

rare. Will not the chromosphere black out the centre again? We shall attempt to answer this question in two ways: firstly, by considering the case in which there is some general absorption and collision-absorption in the chromosphere. For convenience we consider the spectral region in which $J = J_T$ (15).

$\tau=\infty$	$\tau=\tau_1$	$\tau=0$
q_2	q_1	
η_2	η_1	

We divide the atmosphere into two layers with different values of ϵ and k. The equation (16),

$$\frac{d^2(J' - J)}{d\tau^2} = q^2(J' - J) \quad . \quad . \quad . \quad (8.1)$$

breaks up into two parts

$$\frac{d^2(J' - J)}{d\tau^2} = q_1^2(J' - J), \quad 0 < \tau < \tau_1 \quad . \quad . \quad (8.11)$$

with the boundary condition $J' = 2H'$ at $\tau = 0$;

$$\frac{d^2(J' - J)}{d\tau^2} = q_2^2(J' - J), \quad \tau_1 < \tau < \infty \quad . \quad . \quad (8.12)$$

with the boundary condition that J' does not become infinite when $\tau \to \infty$. In addition we have two boundary conditions at $\tau = \tau_1$, given by the continuity of J' and H'. The appropriate solutions are

$$J' = J + Ae^{-q_1\tau} + Be^{q_1\tau} \quad 0 < \tau < \tau_1 \quad . \quad . \quad (8.21)$$

and

$$J' = J + Ce^{-q_2\tau} \quad \tau_1 < \tau < \infty \quad . \quad . \quad . \quad (8.22)$$

Continuity of J' gives

$$Ae^{-q_1\tau_1} + Be^{q_1\tau_1} = Ce^{-q_2\tau_1} \quad . \quad . \quad . \quad (8.31)$$

Since

$$H' = \frac{1}{3(1 + \eta)} \frac{dJ'}{d\tau} \quad \text{and} \quad J = H(2 + 3\tau),$$

continuity of H' gives

$$\frac{1}{1 + \eta_1}\{3H + q_1(-Ae^{-q_1\tau} + Be^{q_1\tau})\} = \frac{1}{1 + \eta_2}\{3H - q_2Ce^{-q_2\tau}\} \quad (8.32)$$

The condition $J' = 2H'$ when $\tau = 0$ gives

$$2H + A + B = \frac{2}{3(1 + \eta_1)}\{3H - q(A - B)\} \quad . \quad . \quad (8.33)$$

We can eliminate A, B, and C from these equations and determine $\frac{H'}{H}$. Here we are concerned with the case where η_1 and η_2 are very large.

We suppose that q_1 and q_2, though small compared with η_1 and η_2, are large compared with unity. (8.33) then becomes

$$A + B = -2H \quad . \quad . \quad . \quad . \quad (8.41)$$

and from (8.31) and (8.32)

$$\frac{Ae^{-q_1\tau_1} - Be^{q_1\tau_1}}{Ae^{-q_1\tau_1} + Be^{q_1\tau_1}} = \frac{\eta_1 q_2}{\eta_2 q_1} \quad . \quad . \quad (8.42)$$

Then from (8.21)

$$r(\nu_0) = \frac{H'(\nu_0)}{H} = -\frac{q_1}{3\eta_1}\frac{A - B}{H} \quad . \quad . \quad . \quad (8.5)$$

Note that if $\tau_1 = 0$ (8.42) becomes

$$\frac{A - B}{A + B} = \frac{\eta_1 q_2}{\eta_2 q_1},$$

and (8.5) becomes

$$r = \frac{2}{3}\frac{q_2}{\eta_2} \quad . \quad . \quad . \quad . \quad . \quad (8.51)$$

which is (7.11), the result for a uniform layer—as it should be.

Eliminating A and B, we get

$$r = \frac{2q_1}{3\eta_1} \cdot \frac{\eta_2 q_1 + \eta_1 q_2 - (\eta_2 q_1 - \eta_1 q_2)e^{-2q_1\tau_1}}{\eta_2 q_1 + \eta_1 q_2 + (\eta_2 q_1 - \eta_1 q_2)e^{-2q_1\tau_1}} \quad . \quad (8.52)$$

which shows that

$$r = \frac{2q_1}{3\eta_1} \quad . \quad . \quad . \quad . \quad . \quad (8.53)$$

if $q_1\tau_1$ is large. Formulæ (8.51)–(8.53) exhibit the way in which the central intensity is determined by the upper layers of the atmosphere.

According to Unsöld (17), the chromosphere has a mean pressure of about $4 \cdot 10^{-9}$ dynes per sq. cm. and contains about 10^{13} atoms of calcium. There is no need to estimate k, since

$$q_1\tau_1 = \sqrt{3\epsilon_1}\eta_1 k\rho x = \sqrt{3\epsilon_1} \cdot l_1\rho x.$$

Taking the average life of a Ca^+ atom as 10^{-8} sec., we find $\sqrt{(3\epsilon_1)}$ is of order 10^{-6}, l_1 is of order 10^{10} (§ 3), and ρx = about 10^{-10} grams per sq. cm. Then $q_1\tau_1$ is of order 10^{-6} and the chromosphere does not blacken the line.

This method of determining the effect of the chromosphere is, however, unsatisfactory, as we can hardly attribute general absorption to a layer in which the pressure is so small. We will therefore reconsider the problem by treating the upper layer as a purely scattering layer—that is, as a Schuster atmosphere in which k and ϵ are both zero.

From Eddington's equation (4), (16), putting $k = 0$ and $\epsilon = 0$, we get

$$\cos\theta\frac{dJ_\nu'(\theta)}{\rho dr} = -lJ_\nu'(\theta) + lJ_\nu',$$

whence

$$\frac{dH_\nu'}{\rho dr} = 0 \quad \text{and} \quad \frac{dK_\nu'}{\rho dr} = -l_\nu H_\nu'.$$

Writing $\sigma_\nu = -\int l_\nu \rho dr$, and using $K = \frac{1}{3}J$, we find

$$J_\nu' = H_\nu'(2 + 3\sigma_\nu)$$

as the equation of the Schuster atmosphere with this set of approximations. We now consider two layers, the upper having no general or collision absorption and the lower layer having coefficients ϵ, k.

$\tau=\infty$	$\tau=0$
η	σ
q	$\sigma=\sigma_1$

In the lower layer we have

$$J' = J + Ae^{-q\tau}$$

$$H' = \frac{1}{3(1+\eta)}\frac{dJ'}{d\tau},$$

and in the upper layer

$$J' = H'(2 + 3\sigma).$$

At the boundary $\sigma = \sigma_1$ we have

$$J' = H_0'(2 + 3\sigma_1)$$

$$H' = H_0'$$

and $J = H(2 + 3\tau)$ for frequencies outside the line which are unaffected by the upper layer. From the continuity of J and H,

$$2H + A = H_0'(2 + 3\sigma_1)$$

$$\frac{1}{3(1+\eta)}\{3H - qA\} = H_0'.$$

Eliminating A, we get

$$r(\nu_0) = \frac{H_0'}{H} = \frac{\frac{2q}{3\eta}}{1 + \frac{2q}{3\eta}\left(1 + \frac{3}{2}\sigma_1\right)}. \qquad (8.6)$$

which shows the effect of the upper layer.

Again using Unsöld's estimate (18) of the number of atoms above 1 sq. cm. in the chromosphere ($= 10^{13}$), we find, on allowing for pure temperature Doppler effect (with a temperature of 5000°), $\sigma_1 = 5{\cdot}5$. If we adopt the structure of the chromosphere suggested in § 6, we find $\sigma_1 = 0{\cdot}5$. In both cases the chromosphere produces an appreciable effect and modifies the pressure to be attributed to the lower layer.

The results are given in the following table, where p_1 is the pressure in the lower layer, allowing $\sigma_1 = 5{\cdot}5$, and p_2 is the pressure in the lower layer, allowing for turbulent motion $\sigma_1 = 0{\cdot}5$. Perhaps the degree of turbulence to be attributed to the chromosphere lies between these two values.

$r(\nu_0)$.	p_1 dynes per sq. cm.	p_2 dynes per sq. cm.
0·05	2480	122
0·03	480	67
0·01	260	22

Again compare with Milne's value, $p = 465$ dynes per sq. cm. at a depth $\tau = \frac{1}{3}$. These figures are satisfactory, but unless we admit the value p_2 the theory suggests that Minnaert's observed value $r(\nu_0) = 0{\cdot}07$ is too high. But it is worth noticing that the effect of the chromosphere does not extend as far from the centre as $|\nu - \nu_0| = 10^{10}$, and that a resolving power greater than 100,000 would be required to detect it.

§ 9. We pass on to consider the principal lines of Ba^+ at λλ4554 and 4934. These two lines correspond exactly to K and H of Ca^+ respectively. Unsöld (18) states that there are $4{\cdot}10^{15}$ atoms of Ba^+ above 1 sq. cm. of the photosphere, as compared with $2{\cdot}3\ 10^{19}$ of Ca^+. The lines, whose contours have been published by Unsöld (4), are very narrow and the central intensity shown is approximately $r(\nu_0) = 0{\cdot}5$ for both lines. (Thus they cannot be fitted to a Schuster contour at $r = \frac{1}{2}$ in the usual way.) Ba^+ has not been observed at great heights in the chromosphere. The following heights in kilometres are given by Stratton and Davidson (19) :—

	Mitchell.	Stratton and Davidson.	
Ba^+	1200	2000	(λ4554)
	750	1500	(λ4934)
Ca^+	14,000	9200	

So that in discussing the central intensities of these lines we can neglect the chromosphere.

Proceeding as for H and K, we find the following values for the scattering constants :—

λ4554	$f = 2/3$	$s_0 = 2{\cdot}9\ 10^{12}$	$a = 1{\cdot}17\ 10^{-7}$	$s_0' = 1{\cdot}17\ 10^{10}$
λ4934	1/3	$1{\cdot}7\ 10^{12}$	$1{\cdot}38\ 10^{-7}$	$6{\cdot}36\ 10^{9}$

Since $N = 4{\cdot}10^{15}$, we get at once $r^*(\nu_0) = 9{\cdot}4\ 10^{-5}$ allowing for pure temperature Doppler effect. This shows that the Schuster contour is hopelessly at fault at the central intensity even in the case of a very narrow line.

Suppose that the value $N = 4{\cdot}10^{15}$ was obtained by fitting the observations to a Schuster contour at $r = 0{\cdot}75$ (we have remarked above that it cannot be done at $r = 0{\cdot}5$ for these lines). This gives $(\Delta\nu)_{3/4} = 1{\cdot}39\ 10^{10}$ sec.$^{-1}$ (λ4554).

Now substitute in (5.1), noticing that for these lines $a_\nu \doteqdot b_\nu$ so that

$$r_\nu = \sqrt{\frac{\kappa}{\kappa + s_\nu}}.$$

Making the fit at $r = \frac{3}{4}$ we get $k = 3{\cdot}7\ 10^5$,† using $s'_0 = 2{\cdot}62\ 10^9$ we get $r(\nu_0) = 5{\cdot}6\ 10^{-3}$. Again, the central intensity given by Milne's formula is much too small even in the case of very narrow lines. Notice that we cannot apply the results of § 6 to Ba^+ as this element is not ejected into the chromosphere.

In applying (7.1) to the Ba^+ lines we cannot use the approximations used in § 7. But, assuming a trial value for ϵ, we can solve (7.1) for k, using the known value of l_ν at $r = \frac{3}{4}$. Then from the known value of $l_0 = s'_0$ we can find $r(\nu_0)$. Corresponding to any chosen ϵ we can calculate the pressure p, assuming that the average life of an excited Ba^+ atom is 10^{-8} sec.

The following table gives some results :—

p_1 dynes per sq. cm.	ϵ.	k_1	$r(\nu_0)$.
$7{\cdot}27\ 10^2$	0·001	$6{\cdot}3\ 10^5$	0·04
$7{\cdot}27\ 10^3$	0·01	$6{\cdot}3$,,	0·10
$7{\cdot}27\ 10^4$	0·1	$6{\cdot}2$,,	0·77

These central intensities should be shown by an instrument of resolving power 10^5, since for $\left|\frac{\nu - \nu_0}{\nu_0}\right| = 10^{-5}$ we have $l_\nu = \frac{1}{2}l(\nu_0)$ approximately (see § 3), and the central intensity is not sensitive to small changes in l_ν. (Computation gives no differences, to the order of accuracy given in the table.)

According to the present view (Eddington's contour) the breadths of spectral lines are determined by the relative numbers of the atoms concerned in their formation, while the central intensity depends on their mean pressure. Thus in the case of the Ba^+ lines the small width indicates a small proportion of Ba^+ in the reversing layer, and the relatively high central intensity indicates that the Ba^+ lines are formed deep down. It may therefore be possible to correlate central intensities of elements with observed heights in the chromosphere, when more reliable data are available.

As in the case of calcium, Eddington's contour gives results which are in much better agreement with the available data than does either of the other two contours proposed.

† A remark on the way in which Milne's κ occurs in these formulæ may not be out of place. The formula is really derived by using a value of s'_ν, which neglects the existence of all the other elements in the atmosphere : hence the value of κ found applies to a pure barium atmosphere : that is, an atmosphere consisting only of Ba^+ would have to possess this value of κ to account for the observed width of the line. Putting in the other elements reduces the value of s_ν, since it is now the fraction scattered by one gram of the mixture. The observations determine the ratio s_ν/κ, so that the value of κ to be attributed to the heterogeneous atmosphere is decreased. The value of κ found by considering the atmosphere as consisting of one element only determines the proportion of this element in the actual atmosphere, for which $\kappa = 10$ (thus the proportion by mass of Ba^+ is about $10/5 \cdot 10^5 = 2{\cdot}10^{-5}$). In this way the relative number of atoms above the atmosphere can be determined.

Conclusion.

By applying H. A. Wilson's formula for the coefficient of line scattering to the centre of the K line in the Sun, we have found that the Schuster and Milne atmospheres give results which are out of agreement with observation of the central intensities. The application to Eddington's model, which differs from Milne's by having an additional type of absorption, is satisfactory and shows that we must attribute the high central intensities observed to the effect of collisions. An upper layer in which no collisions occur does not black out the line again completely, but causes us to attribute slightly higher pressures to the lower layer. These pressures are in good agreement with those found by Milne from other considerations. Similar results are given by the principal lines of ionised barium.

In conclusion I wish to thank Professor Eddington for valuable advice and suggestions in connection with these problems.

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Gonville and Caius College, Cambridge,
1929 *November* 8.

The Normal Wave-lengths of the Calcium Lines H *and* K, *and the Relativity Shift of these Lines in the Prominences and Chromosphere.* By J. Evershed, F.R.S.

The wave-lengths of the lines H and K in the arc in air referred to standard iron lines were determined by St. John in an exhaustive series of measures of arc, spark, and furnace spectra. Fairly consistent results were obtained, but the furnace spectra yielded values 0·002A smaller than the arc spectra.

St. John summarised his results as follows: "The wave-lengths of

the H and K lines of calcium are 3968·476 and 3933·667, as determined from the secondary standards of Fabry and Buisson, with an uncertainty of not more than 0·001A" (*Astrophysical Journal*, **31**, 156, 1910).

In my estimates of the shifts of these lines in the prominences, I have been led to suspect that the above values do not represent the wave-lengths in the combined iron and carbon arc, where the calcium lines are shown as narrow emission lines with a relatively weak iron spectrum. The beautiful definition of the arc lines of iron and calcium shown on the different series of solar spectra obtained with the 6-inch prisms of my spectroheliograph has enabled me to use the measures of these plates for the purpose of determining with accuracy the wave-lengths of H and K referred to the nearest strong iron lines.

The scale of these plates is approximately 1 mm. to the angstrom, and is derived from measures of some or all of the following *Fe* lines :—

3920·261	3930·300	3940·884
3956·682	3969·262	3977·746

These wave-lengths are from the secondary and tertiary standards of the iron arc in air, published in the *Transactions of the International Astronomical Union*, vol. i. These have been used throughout, although revised values have been published in vol. iii. (1928). The correction needed for this latest revision is − 0·001A, which will be applied in the final results.

Within the range of wave-lengths measured, the curve giving the relation of wave-length to measurement is practically a straight line, and the factors for converting measured intervals into wave-lengths can be readily derived from the graphs with the necessary accuracy. The wave-length of K has been derived from the interval 3930 − K, and of H from the interval H — 3969. In some cases where the line 3969 is too broad for accurate measures, the line 3967·424 has been substituted. These intervals correspond with about 3 mm., 0·8 mm., and 1 mm. respectively. Errors of scale will not enter appreciably into such small intervals, and such errors are always much smaller than the accidental error in setting the micrometer thread on a line.

In determining the wave-length of K in a photograph of the combined iron and calcium arc, there is a tendency to obtain low values, due to the presence of a faint iron line very close to K on the violet side. According to my measures, this has the wave-length 3933·605, and it gives trouble when K is relatively weak and the iron spectrum is strong enough to show the line at 3932. According to St. John, there is a weak iron line on the violet side of H also, but this line does not appear on any of my plates, even those of very high dispersion and with a strong iron spectrum. In order to make certain that the iron line, which I call K′, has no influence in my results, I have rejected all plates in which the iron spectrum is relatively strong ; and in addition I have obtained a series of arc spectra with the high-dispersion cinnamate spectrograph which have a scale of 2 mm. to the angstrom. These consist of the arc spectrum of iron alone, and carbon alone, the latter giving the calcium and aluminium lines with a weak *Fe* spectrum. In some of the iron spectra,

which show the calcium lines very faintly, K and K′ are clearly resolved, and have been measured separately, these spectra obtained with the liquid prism containing ethyl cinnamate being also of excellent definition and high resolving power. The wave-lengths from this series of spectra confirm those obtained with lower dispersion. Finally, I have measured a series of high-dispersion plates of the iron and calcium arc photographed at Kodaikanal with a large Anderson grating in the third and fourth orders.

The variations from plate to plate, which may differ from the mean value by at the most 5 units in the third decimal, are larger than would be expected from the accuracy with which a line can be bisected; and it is curious that these variations are as large for the high-dispersion plates as for those having only one-half the scale. This seems to indicate real variations of wave-length in the lines themselves, and it is possible that the so-called "stable lines" are subject to very small changes near the poles of an arc. H and K are not subject to pole effect, and the reference lines of iron are all classed as stable. The iron spectra are from a Pfund arc carrying 6 amperes at 100 volts, but the length of arc varied considerably, and in many cases light from the region near the poles entered the slit. Some of the cinnamate spectra, however, represent the centre of a long arc, and these yield more consistent values than the spectra where precautions were not taken to exclude the regions near the poles. This applies more especially to H, which always gives larger variations than K.

The K line is measured from the narrow emission line at 3930, or from the equally narrow reversal of the broader emission line. This absorption line, representing the outer region of the arc, can scarcely be affected by the poles. The calcium lines are always measured as emission lines. Whatever these apparent variations may be due to, the only course is to treat them as accidental errors, or to repeat measures where a mistake is suspected. In nearly all cases of repetition the second measure agrees with the first, within 0·002A.

To save printing long lists of figures, I give only the mean results from the four series of plates, with their probable errors derived from the residuals of the individual values. These are as follows :—

Wave-length of K *referred to the Iron Line at* 3930·300.

25	prominence and arc spectra . . .	3933·6643 ± ·0001
25	chromosphere and arc spectra . . .	·6639 ± ·0003
23	cinnamate arc spectra	·6638 ± ·0003
9	grating arc spectra	·6639 ± ·0002

Wave-length of H *referred to the Iron Lines at* 3967·424 *and* 3969·262.

35	prominence and arc spectra . . .	3968·4706 ± ·0003
35	chromosphere and arc spectra . . .	·4715 ± ·0003
20	cinnamate arc spectra	·4700 ± ·0004
9	grating arc spectra	·4717 ± ·0005

The agreement of the different values of K is extraordinarily close, and justifies the retention of the fourth decimal, but this of course only implies that the interval between the iron and calcium lines shows this degree of accuracy, the absolute position of the iron line being known only to the third place. The mean results to three places are as follows :—

K 3933·664 H 3968·471.

These values are smaller than those of St. John, by 0·003A in the case of K, and 0·005A in the case of H. It is not easy to account for these differences. Fabry and Buisson's wave-lengths of standard iron lines do not differ appreciably from the more recent values, but St. John's results appear to depend on very large measured intervals, which may have introduced errors of scale or of the micrometer screw.

According to my measures, the wave-lengths of K and H in the carbon-iron arc in air, corrected to the recently revised values of the iron lines, are as follows :—

K 3933·663 H 3968·470.

The Relativity Shift of H *and* K *in the Prominences and the Chromosphere.*—In *Monthly Notices*, **89**, 254, 1929, estimates are given of the shift towards red of the H and K lines in the prominences. The mean value of the whole series of 289 spectra, corrected for the pressure effect of the arc in air, is + 0·0154A, and the mean for a selected series of 180 spectra is + 0·0142A. As H and K yield sensibly the same shift, both lines are included in the means.

A previous estimate from 92 prominence spectra obtained in 1927 gave a mean value + 0·0109A. In the earlier series I used the *Fe* arc lines for determining the positions of the calcium lines in the prominences, and assumed that the normal wave-lengths of K and H were St. John's values 3933·667 and 3968·476 respectively. In the later series I measured the direct displacements between the solar and terrestrial calcium lines, using a carbon arc which gave strong but very narrow H and K lines, with a weak or absent *Fe* spectrum.

The difference of about 0·004A in the two series could be accounted for by supposing that the assumed values of H and K were too large by this amount, and the foregoing measures appear to show quite definitely that this is the case. The prominence measures are therefore brought into good agreement and indicate that the shift towards red is approximately + 0·015A.

I have now to add some measures of the shift of these lines in the chromosphere at the sun's limb. Here the lines are much wider and more difficult to measure than in the prominences. I have nevertheless been able to secure a series of plates which yields fairly reliable results. The spectra are taken in pairs at the east and west points on the sun, each pair giving the rotation shift and the general shift towards red. The mean values of the shift from 22 such pairs are again sensibly the same for K and H, K giving + 0·0133A, and H + 0·0135A. Taking

the mean of these, and adding the pressure effect of the arc in air, the result is as follows :—

Observed mean shift of H and K . .	+ 0·0134 ± ·0004
Pressure effect	+ 0·0017
Chromosphere – arc in vacuum	+ 0·0151

This agrees well with the prominence result.

The Einstein relativity effect for the mean of H and K is + 0·0081A at 30″ above the limb, and + 0·0083A for the chromosphere. The excess over the Einstein effect is therefore about + 0·007A for prominences and chromosphere, and any explanation of the cause of this discrepancy which might apply to the conditions in the chromosphere must be equally applicable in the region of the prominences altogether outside the sun's atmosphere.

Stellar Parallaxes determined photographically at the Cape Observatory. (*Second List.*) By H. Spencer Jones and J. W. Jackson.

The first list of stellar parallaxes derived from photographs obtained with the Victoria Telescope at the Cape Observatory was published in *M.N.R.A.S.*, **89**, 497, 1929, to which reference should be made for general details as to methods of observation and reduction.

The stars in the present list are numbered in rotation, in continuation of the first list. The Durchmusterung number of the star is given as in the Draper Catalogue, *i.e.* the B.D. number down to Dec. −23°, the Co.D. number south of this limit to −52°, and the C.P.D. number south of −52°. The magnitudes and spectral type are from the Draper Catalogue. The first value of the proper-motion for each star is that given by meridian observations, by comparisons of plates in the blink-microscope or by other methods ; many of these values are subject to a large probable error. The second value of the proper-motion in R.A. is that derived from the photographs. In the last column, the first value of the probable error given for any star refers to the parallax and the second value to the proper-motion.

The parallaxes are relative values. The mean magnitude of the comparison stars averages about 11^{m}. The average probable error for a plate of unit weight in this list is ± 0″·024. The probable errors assigned to the parallaxes and proper-motions are based upon the probable error of unit weight derived from the internal agreement of each series.

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001.	Proper-Motion. Dec. Unit ″·001.	P.E. Unit Weight. Unit ″·001.	Paral-lax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
102	37 Phœnicis	5·77	Go	0 1 8	+ 530	− 43	25	+ 58	± 8
	−49° 14337	6·3		−49° 37′·9	+ 511				± 9
103	−53° 42	6·96	Go	0 11 54	+ 234	+ 200	21	+ 64	± 7
		7·2		−53° 12′·7	+ 307				± 6
104	−37° 273	7·67	Go	0 42 45	− 214	− 360	24	+ 29	± 8
		8·4		−37° 28′·6	− 198				± 7
105	−31° 455	8·2	Ko	1 4 12	+ 255	− 163	20	+ 36	± 7
		9·4		−31° 27′·6	+ 200				± 7
106	−42° 432	8·6	Go	1 9 56	− 299	− 100	21	+ 38	± 7
		9·0		−42° 25′·3	− 134				± 7
107	−27° 430	8·3	Go	1 13 57	+ 261	+ 209	30	+ 25	±10
		9·0		−27° 2′·0	+ 219				± 8
108	− 9° 256	8·9	Go	1 14 1	− 237	− 471	21	+ 34	± 7
		9·5		−9° 27′·2	− 237				± 6
109	−28° 433	8·00	Go	1 20 4	+ 242	− 315	29	+ 68	±10
		8·8		−28° 21′·1	+ 308				±10
110	−24° 658	6·92	Ko	1 28 30	+ 268	− 167	27	+ 64	± 9
		7·9		−24° 41′·2	+ 234				±10
111	−21° 288	7·8	Go	1 39 37	− 49	− 312	23	+ 31	± 8
		8·9		−21° 5′·4	− 69				± 8
112	−16° 301	8·5	Go	1 40 24	+ 278	− 179	22	+ 15	± 8
		9·1		−16° 23′·6	+ 272				± 6
113	−41° 556	7·3	Go	1 56 18	+ 384	− 440	28	+ 54	± 9
		8·0		−41° 12′·2	+ 435				± 9
114	−24° 889	9·5	G5	2 0 58	+ 402	− 48	20	+ 16	± 7
		9·5		−24° 51′·4	+ 368				± 7
115	−24° 891	9·2	G5	2 1 2	+ 407	− 40	24	− 2	± 9
		9·5		−24° 51′·2	+ 404				± 9
116	−31° 942	9·4	G5	2 16 58	+ 90	− 320	25	+ 31	± 8
		9·8		−30° 59′·8	+ 131				± 8
117	−31° 943	8·5	Ko	2 16 58	+ 547	+ 96	21	+ 42	± 7
		9·5		−31° 23′·6	+ 559				± 6

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001	Proper-Motion. Dec.	P.E. Unit Weight. Unit ″·001.	Paral-lax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
118	−24° 1106	7·52	Ko	2 25 0	+ 277	+ 32	17	+ 9	± 5
		8·3		−24° 33′·0	+ 302				± 6
119	−17° 484	9·5	..	2 26 41	+ 472	+ 10	22	+ 2	± 8
		9·7		−17° 25′·6	+ 403				± 7
120	−30° 990	8·1	Go	2 36 17	+ 590	+ 75	20	+ 39	± 7
		8·1		−30° 34′·0	+ 544				± 7
121	ι Horologii	5·42	F5	2 39 9	+ 301	+ 222	25	+ 73	± 8
	−51° 641	6·8		−51° 13′·9	+ 329				± 9
122	−46° 790	9·4	Ko	2 39 58	+ 10	− 470	24	+ 15	± 9
		10·0		−46° 51′·8	+ 57				± 8
123	−13° 544	6·14	G5	2 47 43	+ 365	− 199	21	+131	± 8
		6·92		−13° 10′·5	+ 402				± 8
124	−33° 992	8·6	G5	2 48 9	+ 421	+ 36	24	+ 36	± 8
		8·8		−33° 51′·7	+ 412				± 7
125	−36° 1091	8·6	K5	2 49 58	+ 496	− 170	26	+ 37	± 8
		9·3		−36° 18′·4	+ 498				± 8
126	ε Fornacis	5·90	G5	2 57 19	+ 284	− 408	26	+ 15	± 9
	−28° 987	6·9		−28° 28′·5	+ 257				± 8
127	3 G Reticuli	..	..	..	..	..	26	+107	± 9
	−63° 217				+1317				± 9
	*	..	..	..	..	..	33	+106	±12
	,, ,,				+1341				±11
	Mean	5·48	Go	3 15 36	+1327	+ 677	..	+107	± 7
		6·5		−62° 57′·5	+1327				± 7
128	ζ Reticuli	..	..	..	..	..	28	+117	±10
	−62° 265				+1322				± 9
		..	..	..	..	..	27	+ 83	± 9
	,, ,, †				+1286				± 9
	Mean	5·16	Go	3 16 2	+1315	+ 692	..	+ 98	± 7
		5·72		−62° 53′·3	+1304				± 6

* Measured also on plates of region 128.
† ,, ,, ,, 127.

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001.	Proper-Motion. Dec.	P.E. Unit Weight. Unit ″·001.	Parallax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
129	−29° 1231	7·34	Go	3 15 50	+ 395	− 85	25	+ 6	± 8
		8·0		−29° 12′·8	+ 326				± 8
130	−31° 1384	7·7	Go	3 22 4	+ 270	+ 231	28	+ 50	± 9
		8·6		−30° 58′·5	+ 170				± 9
131	−48° 1011	9·1	K5	3 31 51	+ 405	+ 295	29	+104	±11
		9·6		−48° 45′·6	+ 384				± 9
132	−28° 1214	8·7	G5	3 32 49	− 103	− 305	24	+ 2	± 8
		9·4		−28° 27′·3	− 85				± 8
133	−11° 716	8·8	Go	3 37 4	+ 319	− 253	21	+ 23	± 9
		9·4		−11° 0′·7	+ 270				± 7
134	−51° 887	7·02	F8	3 40 11	+ 123	+ 510	16	+ 21	± 6
		7·6		−50° 58′·4	+ 109				± 5
135	−28° 1276	7·9	Ko	3 41 13	+ 304	+ 171	21	+ 32	± 7
		8·8		−28° 10′·8	+ 257				± 7
136	−23° 1619	6·80	Go	3 49 38	+ 319	− 314	23	+ 32	± 8
		7·1		−23° 25′·5	+ 344				± 8
137	−36° 1508	7·56	F8	3 51 3	+ 153	+ 363	30	+ 44	±11
		8·0		−36° 0′·1	+ 102				±11
138	−34° 1491	6·67	Go	3 58 13	+ 376	+ 45	27	+ 19	±10
		7·3		−34° 45′·6	+ 350				±10
139	−57° 612	7·9	F5	3 59 34	+ 322	+ 310	26	+ 40	± 9
		8·5		−57° 29′·8	+ 332				± 9
140	−21° 784	9·1	..	4 2 12	+ 112	− 780	25	+ 62	±10
				−21° 6′·0	+ 61				± 9
141	24 G Reticuli	6·44	Go	4 6 16	+ 200	+ 338	24	+ 8	±10
	−64° 305	6·2		−64° 29′·7	+ 163				± 8
142	−53° 672	7·1	Ko	4 13 23	+ 918	+ 340	29	+ 72	± 9
		8·9		−53° 34′·1	+ 753				± 9
143	−25° 1868	7·46	Go	4 19 11	+ 41	− 294	22	+ 30	± 8
		8·9		−25° 37′·2	+ 46				± 8

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001.	Proper-Motion. Dec.	P.E. Unit Weight Unit ″·001.	Parallax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
144	−49° 1366	8·7	Go	4 29 4	− 10	+ 350	30	+ 24	±12
		9·5		−49° 32′·5	+ 26				±10
145	50 Eridani	4·59	Ko	4 29 35	− 104	− 273	24	− 13	± 8
	−30° 1883	5·59		−29° 58′·1	− 125				± 9
146	−65° 361	9·6	Go	4 37 38	+ 662	+1307	20	+ 16	± 7
		10·2		−65° 38′·8	+ 704				± 7
147	−50° 1492	7·77	G5	4 43 7	− 451	− 337	24	+ 57	± 8
		8·0		−50° 14′·7	− 485				± 8
148	−31° 2095	7·87	G5	4 50 54	− 26	− 300	30	+ 4	±10
		8·6		−31° 35′·2	− 11				±11
149	−28° 1839	7·6	K5	4 51 43	+ 197	− 258	20	+ 27	± 7
		8·9		−28° 43′·0	+ 181				± 7
150	−5° 1123	6·50	Ko	4 55 51	+ 552	−1118	23	+ 88	± 9
		7·50		−5° 52′·3	+ 555				± 8
151	2 G Columbæ	6·60	Go	5 5 17	− 87	+ 286	19	+ 28	± 6
	−41° 1727	7·7		−41° 21′·0	− 125				± 7
152	−15° 978	8·0	G5	5 9 49	+ 234	− 211	26	+ 59	± 8
		8·8		−15° 56′·2	+ 189				± 8
153	−77° 192	7·0	Go	5 11 16	+ 144	− 360	22	+ 7	± 8
		7·6		−77° 40′·6	+ 145				± 8
154	o Columbæ	4·91	Ko	5 13 53	+ 80	− 346	23	+ 26	± 7
	−35° 2214	5·91		−34° 59′·6	+ 94				± 8
155	−23° 2865	8·8	Ko	5 30 37	+ 393	− 407	22	+ 70	± 7
		9·1		−23° 31′·3	+ 306				± 7
156	−46° 1936	7·6	Go	5 35 22	− 135	− 470	24	+ 34	± 8
		8·4		−46° 9′·0	− 133				± 8
157	γ Mensæ	5·06	Ko	5 35 50	+ 93	+ 301	23	+ 6	± 7
	−76° 333	7·3		−76° 24′·7	+ 73				± 6
158	−50° 1977	6·50	Ko	5 51 44	000	+ 600	22	+ 38	± 7
		7·2		−50° 23′·7	+ 92				± 7

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001.	Proper-Motion. Dec.	P.E. Unit Weight. Unit ″·001.	Paral-lax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
159	36 G Doradûs	4·53	Ko	5 53 20	+ 132	+ 546	27	+ 10	± 9
	−63° 498	6·0		−63° 7′·2	+ 132				± 9
160	−21° 1377	8·7	K5	6 6 22	− 97	− 670	28	+190	± 8
		9·3		−21° 49′·5	− 149				± 9
161	−22° 1364	6·04	Go	6 12 51	+ 167	− 254	31	+ 14	± 9
		7·7		−22° 40′·3	+ 149				±10
162	−48° 2259	6·39	Go	6 17 28	+ 161	− 300	25	+ 36	± 8
		7·0		−48° 41′·3	+ 249				± 8
163	−25° 3237	6·04	F8	6 23 10	− 208	− 227	27	+ 41	± 9
		6·6		−25° 47′·4	− 192				± 9
164	−27° 3248	6·43	F8	6 40 52	− 29	+ 307	26	+ 58	± 8
		7·1		−27° 14′·8	− 8				± 8
165	72 G CMa	5·92	F8	6 41 38	− 221	− 329	26	+ 34	± 9
	−31° 3640	6·8		−31° 40′·8	− 204				± 9
166	38 G Puppis	5·05	F2	6 47 5	− 15	+ 357	24	+ 58	± 7
	−46° 2703	6·6		−46° 30′·5	− 20				± 7
167	90 G CMa	6·03	Go	6 49 35	+ 290	− 437	21	+ 41	± 7
	−28° 3554	7·0		−28° 24′·2	+ 300				± 7
168	−49° 2676	7·8	G5	7 8 53	000	+ 740	24	+ 12	± 8
		7·8		−49° 16′·7	− 33				± 7
169	−46° 3046	7·6	Ko	7 14 37	− 50	+ 583	19	+ 71	± 7
		7·4		−46° 48′·8	− 32				± 6
170	−34° 3611	6·75	F8	7 23 19	− 286	+ 80	27	+ 16	± 8
		7·4		−34° 6′·8	− 257				± 8
171	Boss 1965	5·94	F8	7 24 49	− 187	− 263	25	+ 58	± 8
	−14° 1925	6·44		−14° 47′·1	− 182				± 8
172	159 G Puppis	5·22	G5	7 39 51	− 72	− 568	26	+ 49	± 8
	−44° 3675	6·0		−44° 55′·2	− 81				± 9
173	9 Puppis	5·34	Go	7 47 8	− 60	− 339	25	+ 74	± 8
	−13° 2267	5·90		−13° 38′·0	− 92				± 9

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001.	Proper-Motion. Dec.	P.E. Unit Weight. Unit ″·001.	Parallax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
174	212 G Puppis	5·02	F2	7 48 32	− 203	+ 257	28	+ 68	± 8
	−34° 4036	6·0		−34° 27′·2	− 219				± 9
175	−29° 5555	6·93	Go	8 2 55	+ 366	− 405	22	+ 38	± 7
		7·7		−29° 6′·3	+ 333				± 7
176	82 G Carinæ	4·80	F5	8 7 21	− 152	− 288	23	+ 50	± 6
	−60° 1074	5·9		−60° 59′·9	− 172				± 7
177	11 G Pyxidis	6·36	G5	8 28 57	−1123	+ 737	23	+ 64	± 7
	−31° 6229	7·4		−31° 10′·9	−1111				± 8
178	Boss 2312	5·13	G5	8 34 45	− 234	+ 416	21	+ 75	± 7
	−22° 2345	6·2		−22° 19′·3	− 262				± 7
179	110 G Carinæ	5·17	Fo	8 56 57	− 184	+ 270	24	+ 49	± 8
	−58° 1327	5·45		−58° 42′·1	− 171				± 8
180	−77° 507	6·9	Go	9 24 50	− 228	+ 370	26	+ 5	±10
		7·5		−77° 28′·1	− 251				± 9
181	−45° 5556	8·58	G5	9 50 35	+ 63	+ 460	24	− 20	±10
		8·5		−45° 15′·5	+ 91				± 8
182	−29° 8875	6·53	Go	11 3 9	− 524	− 154	29	+ 31	±11
		7·6		−29° 37′·8	− 491				±10
183	−29° 9950	7·70	Go	12 45 0	− 436	+ 7	23	+ 18	± 7
		8·5		−30° 2′·0	− 453				± 7
184	−14° 3970	7·86	G5	14 25 49	+ 221	− 372	25	+ 38	± 9
		8·64		−15° 11′·1	+ 237				± 9
185	Boss 3813	5·76	K5	14 51 37	+1039	−1754	22	+172	± 6
	−20° 4125	7·3		−20° 57′·9	+1025				± 7
186	−40° 9847	8·9	G5	15 35 35	− 319	− 110	20	+ 14	± 6
		9·4		−40° 29′·8	− 155				± 7
187	−47° 10376	8·9	Ao	15 44 11	− 151	+ 350	20	+ 3	± 6
		9·0		−47° 57′·4	+ 6				± 7
188	−42° 10934	7·6	K5	15 50 51	− 266	− 250	23	+ 49	± 7
		8·6		−42° 19′·7	− 263				± 8

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001	Proper-Motion. Dec.	P.E. Unit Weight. Unit ″·001.	Parallax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
189	−17° 4461	8·7	Go	15 52 43	− 243	− 200	22	+ 7	± 7
		9·3		−17° 27′·5	− 250				± 8
190	−69° 2521	8·6	F8	16 11 1	000	− 350	27	+ 5	± 8
		9·1		−69° 28′·8	− 19				± 9
191	−27° 10902	7·5	Go	16 12 56	+ 49	− 297	22	+ 48	± 7
		8·1		−28° 2′·6	+ 67				± 9
192	−24° 12671	9·1	Go	16 15 53	− 260	− 142	30	− 3	± 8
		9·2		−24° 45′·0	− 109				±10
193	−13° 4418	8·6	Ko	16 18 45	− 223	− 215	24	+ 49	± 7
		9·6		−13° 24′·2	− 183				± 9
194	−57° 8091	7·01	Go	16 30 39	− 167	− 250	26	+ 34	± 8
		7·5		−58° 2′·9	− 215				± 9
195	−2° 4230	7·09	Go	16 35 55	− 37	− 441	19	+ 45	± 6
		7·65		−2° 39′·1	− 11				± 6
196	Boss 4342	7·90	K5p	16 59 51	− 913	−1147	22	+ 79	± 6
	−4° 4225	9·08		−4° 53′·8	− 948				± 7
197	−35° 11426	6·18	F8	17 9 41	− 171	− 320	23	+ 1	± 6
		7·0		−35° 37′·6	− 121				± 7
198	142 G Scorpii	5·89	K2	17 12 9	+1180	− 179	24	+140	± 7
	−34° 11626	7·1		−34° 52′·7	+1092				± 8
199	Boss 4433	5·34	G5	17 25 15	− 124	− 175	24	+ 42	± 6
	−0° 3300	6·12		−0° 58′·8	− 155				± 8
200	−65° 3528	6·54	F8	17 46 20	000	− 390	19	+ 55	± 5
		6·8		−65° 41′·5	+ 76				± 7
201	−22° 4475	8·8	Go	17 52 56	− 308	− 143	20	+ 23	± 6
		8·8		−22° 22′·3	− 288				± 7
202	−33° 12962	8·6	G5	18 6 59	− 208	− 264	22	+ 6	± 6
		9·1		−33° 31′·0	− 133				± 7
203	−1° 3500	8·22	K5	18 24 40	+ 142	− 208	18	+ 42	± 6
		9·40		−1° 52′·7	+ 181				± 6
204	262 G Sagittarii	5·67	F5	19 53 54	+ 133	− 310	19	+ 53	± 5
	−34° 14082	6·8		−33° 58′·1	+ 114				± 6

No.	Name and D.M. No.	Mag. Vis. Phot.	Type.	R.A. Dec. (1900).	Proper-Motion. R.A. Unit ″·001	Proper-Motion. Dec.	P.E. Unit Weight. Unit ″·001.	Parallax. Unit ″·001.	P.E. Unit ″·001.
				h m s					
205	Boss 5119	5·91	F8	19 54 21	− 285	− 395	21	+ 42	± 7
	−10° 5238	6·41		−10° 13′·2	− 246				± 7
206	5 G Capricorni	5·69	Ko	20 9 3	+1243	− 192	23	+ 94	± 6
	−27° 14659	7·2		−27° 19′·9	+1267				± 7
207	109 G Pavonis	5·30	F8	20 31 46	+ 299	− 573	32	+ 47	±10
	−60° 7419	5·9		−60° 53′·0	+ 270				±12
208	−44° 14214	6·64	Go	20 51 3	− 564	− 988	26	+ 45	± 8
		7·0		−44° 29′·2	− 508				±10
209	−61° 6537	6·72	Go	21 10 47	+ 440	− 430	21	+ 40	± 7
		7·4		−61° 45′·5	+ 436				± 7
210	−39° 14192	6·65	Map	21 11 24	−3331	−1172	26	+253	± 8
		7·9		−39° 15′·2	−3262				± 8
211	−56° 9645	8·6	G5	21 19 37	+ 694	+ 180	22	+ 32	± 6
		9·9		−56° 33′·6	+ 655				± 6
212	−53° 10224	7·16	F8	21 54 58	000	− 490	26	+ 15	± 8
		7·6		−53° 33′·6	− 6				± 9
213	−32° 17321	6·46	K5	22 50 50	+ 318	− 190	25	+118	± 8
		7·4		−32° 5′·7	+ 307				± 9
214	88 G Gruis	5·78	F8	23 4 24	− 344	+ 25	22	+ 22	± 7
	−43° 15281	7·0		−43° 24′·1	− 300				± 8

Three Features of the Lunar Crust, from Photographs taken at Dedham.

By H. G. Tomkins, C.I.E., F.R.P.S. (Plate 6.)

Two years have elapsed since I was able to report the completion of my 24-inch Cassegrain reflecting telescope, and to outline the work which it was proposed to undertake with it on the lunar surface. Since then over 80 plates have been obtained, and, as was expected would be the case, it has been possible to co-ordinate some of the information gleaned from the individual plates taken under various lightings and librations. The results are described below, the first being perhaps the most important, though it is difficult to make a comparison in view of the extensive phenomena which are indicated. For want of a better name I call it a "flow," which indicates its nature without unduly presupposing its origin. The origin of this disturbance is situated in the Mare Nectaris over an area of about 27,000 square miles. This *mare* seems to have originated on the eastern side of the floor, and, judging

from the traces that have been left, disturbances recurred no less than five times, breaking their banks on each occasion and leaving a succession of scars until its present limits were finally reached. On the fourth occasion the disturbance reached and damaged the northern wall of Fracastor, and on the fifth it invaded that formation over about half its floor, the flow mark forming part of the last ring-wall, which still stands. That the substance of the disturbance was some kind of fluid or plastic matter there can be little doubt, and that it broke its bounds between the present crater Theophilus and Isodorus, the flow passing by Mädler out towards the Mare Tranquillitatis, is evident. Just north of Theophilus the flow seems to have been joined by another flood issuing from Catharina and Cyrillus, the northern walls of both of which have been broken through. Evidence of this stream is seen north of these craters and just north-east of Theophilus.

The flow then breaking down or submerging everything in its path, proceeded into the Mare Tranquillitatis. Its course is very well marked, and on reaching the formation Jansen the stream divided, one portion proceeding east towards the Hæmus Mts. and the other north-west. Taking the former first, the flow has met with an obstacle in the Hæmus Mts. and the region just south of them, and has eroded and broken up both. The photographs also show that though the flow has partially flooded the region south of Julius Cæsar, it has in addition cut through the high ground north of that formation. It has also broken through into the margin of the Mare Serenitatis between Plinius and Menelaus. The flow can now be traced through the Hæmus Mts., which it has thoroughly eroded up to the western side of the Apennine Mts. The approximately parallel formations in this part of the Moon have long been known and difficult to explain, and among other observers I brought them to notice in the year 1908. The photographs now seem to account for them, and to show the path of the flow right into the Apennine range. A small flooding has also occurred on the west side of the range. The flow does not end its course, however, in the Apennine Mts., but has even cut its way through this enormous range, and issued on to the plain on the other side. The cuttings are quite clear and the emergence of the flow on to the plain on the eastern side of the range can be seen in three streams. In order to make this matter sure I observed the range visually with the 24-inch on 1929 November 10. Libration was very favourable, and I secured a good view of two of the channels right through the range. As Mr. Bennett lives within reach of my telephone I rang him up, and he very kindly verified my observations at once on the same evening at his observatory, thus leaving no doubt whatever on the point.

It will be seen that the flow extends farther to Archimedes, which divides it, and it is then lost in the Mare Imbrium. Returning now to the other branch of the flow in the Mare Tranquillitatis, it will be seen that it proceeds from Jansen north-west until it has met the high ground on which stands Proclus, and has then flooded part of this region and found no escape. It has therefore turned away north-east and proceeded between the craters Dawes and Vitruvius and then down the north-

western border of the Mare Serenitatis, skirting the rampart of Posidonius and cutting into the higher ground behind that formation, which it has also flooded. It has then made a channel for itself by Eudoxus and Aristoteles and on between Plato and Fontenelle to high ground, where it has finally terminated.

The whole of this great flow consists therefore of a main stream from Mare Nectaris which has been divided by the Mare Serenitatis into two branches, one passing on the south and the other on the north round that sea until stopped by higher ground. That it took some stopping, however, is evident from its having cut through the enormous Apennine range of mountains.

The length of the course of this gigantic flow is approximately 1400 miles for the Apennine branch and 2000 miles for the Posidonius portion, each being reckoned from the Mare Nectaris.

I come now to the second feature. This is apparently a system of fissure eruptions proceeding from the crater Kepler. It consists of a single fissure arising in Kepler and proceeding through Hortensius and Reinhold to Lalande, where it throws out two branches, and then itself goes on south-west and after passing between Ptolemäus and Alphonsus and up by the west wall of the latter, turns west to Airy. It then goes on to Abulfeda, and apparently ends at a spot near Kant.

The second branch after leaving Lalande passes through Herschel, and after touching the north-west wall of Ptolemäus goes through Halley and Hind, on to Alfraganus and then to Hypatia, after which it seems to disappear. But it is rather significant that if the fissure were continued further it would pass through the southern end of Mare Tranquillitatis by Torricelli, and if still further extended would lie almost exactly on the twin craters Messier and Messier A, and along the two rays from those formations. I should not like to say, however, that this extension exists, though on some of my plates there is some suggestion of it.

The third branch of the formation after leaving Lalande passes through Mosting A and Rhæticus to Godin, involves also Agrippa, and then goes through Sosigenes, Maclear, and Ross on to the Mare Serenitatis, where it ends in a row of about five tiny craterlets, but it does not reach Vitruvius. It is interesting to note that this latter part from Sosigenes lies across a portion of the flow I have already dealt with. This would indicate that these fissure eruptions are younger than the flow.

I now come to my third feature, which is a much smaller matter, but in some ways not less important. It consists of a narrow bright line or ray from Kepler which divides just before reaching Copernicus. One arm passes directly through the walls and centre of Copernicus, issuing beyond Copernicus on the west, and from the indications on the plates possibly extends across the Sinus Æstium and a little farther. The other arm of the ray passes just to the north of Copernicus, cutting and wrecking the foot of the wall and the ground thereabout. It then passes in a curved sweep down towards Archimedes and into the rough ground in that region. It may even proceed

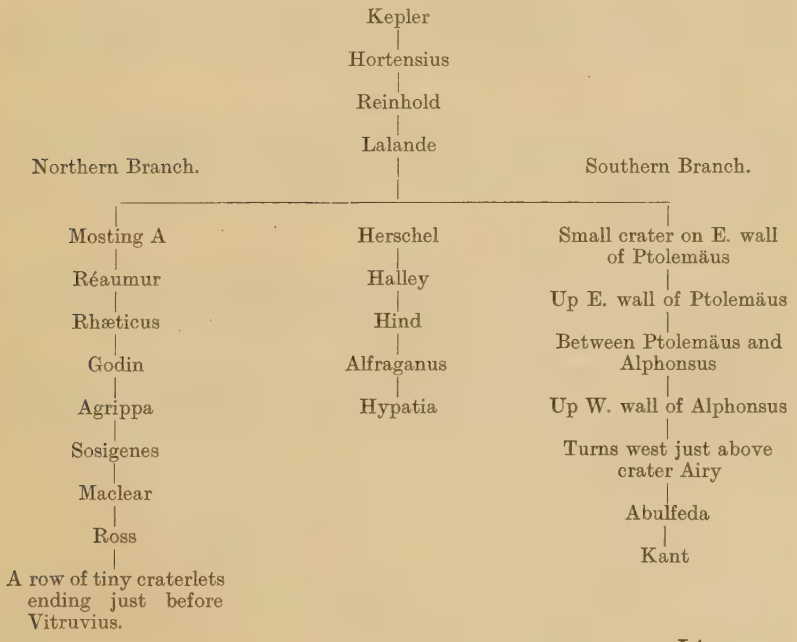
MAP OF THE MOON
MARE
SERENITATIS
PRIM
PALOS
OCEANUS

round the end of the Apennines, but this is doubtful. There is a very bright spot or perhaps a craterlet just outside the east wall of Copernicus on the ray near where the latter divides. I observed the ray visually on 1929 October 27, and found a slight gap in the top of the western wall of Copernicus which would correspond with the course of the ray. The foot of the wall of Copernicus outside also appeared to be broken where the branch passed it.

It is rather early to theorise on these results so far obtained from the plates, but two points seem to emerge. First, the features appear to indicate an igneous and internal origin; and secondly, they give us the relative age of some of the lunar formations. The fact that the ray from Kepler has cut through Copernicus suggests that the former crater is younger than the latter. The fissure eruption similarly indicates its age as compared with the ages of the craters through which it has cut, and the third arm across the Mare Tranquillitatis and on the flow shows that the fissure is younger than the flow, and that both fissure and flow are younger than the Mare Tranquillitatis. The flow mark across the floor of the crater Fracastor, and the path of the stream by Theophilus and Mädler, also indicate that they are younger than either of those formations.

I have put forward these few results thus early so that Fellows may have some idea of the line of inquiry which I am taking up, and the stage so far reached.

Plan of Fissure Eruption.



On the Nature of Wolf-Rayet Emission. By C. S. Beals, M.A., D.I.C., Ph.D. (Plates 7 and 8.)

Introduction.

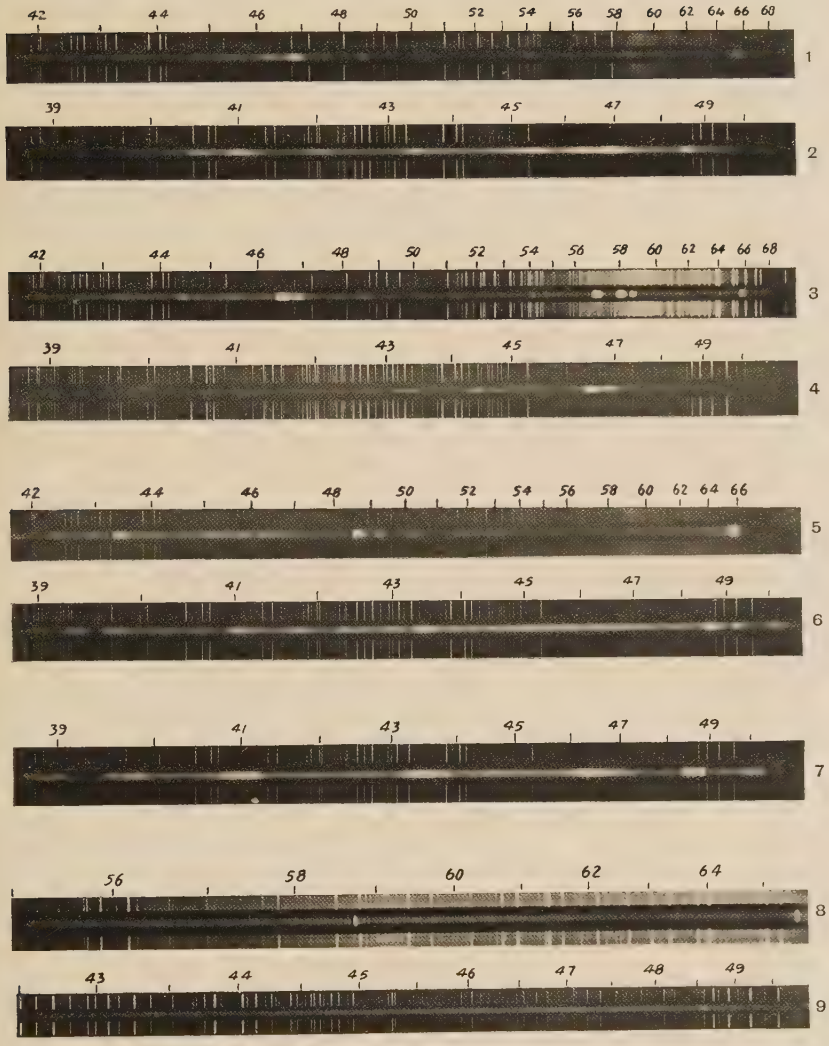
The first three examples of Wolf-Rayet stars were discovered visually by Wolf and Rayet * at the Paris Observatory in 1867. Respighi,† Pickering,‡ and Copeland § have recorded similar discoveries also by visual methods. The Harvard objective prism-plates revealed many more examples, and the majority of Wolf-Rayet stars now known were discovered by the Harvard observers.

The earliest wave-length measurements of emission bands were made by Campbell ‖ in 1892. Wright,¶ in 1918, showed that the nuclei of planetary nebulæ were frequently Wolf-Rayet stars, measured the wave-lengths of bands in a number of these objects, and identified many of them with laboratory wave-lengths. In 1924 J. S. Plaskett ** published the results of an investigation of all the northern Wolf-Rayet stars of magnitude 9·0 or brighter. His work included a detailed description of Wolf-Rayet spectra, measures of wave-length, identifications of bands, and suggestions as to the classification of these stars. Other workers in this field include Perrine †† and Miss Payne,‡‡ who have published data on a number of southern Wolf-Rayet stars.

The most characteristic feature of the spectra of Wolf-Rayet stars is the appearance of atomic emission in the form of broad bands superposed on a comparatively faint continuous background. The appearance of the bands and their contrast with the continuous spectrum is illustrated in Plate 7, where spectra of two typical stars of this type have been reproduced. That this broad band emission is of very different character from that met with in ordinary emission-line stars is evident from an examination of Wolf-Rayet spectra. The bands are from 10 to 100 angstroms in width, their strength, relative to the continuous spectrum, is enormously greater than that of the more usual type of stellar emission, and many of the wave-lengths appearing in Wolf-Rayet stars are not met with in other types of stellar emission spectra.

In attempting to frame a hypothesis to explain this type of emission, attention is naturally focussed on the great width of the bands, since this is the feature which differentiates Wolf-Rayet stars most sharply from other emission-line objects. The most promising explanation appears to be that the broadening is a Doppler effect, and that Wolf-Rayet emission is therefore associated with large velocities of the emitting gases in the line of sight. It is extremely unlikely that other known causes of broadening, such as the Stark effect, the Zeeman effect, or the effect of high pressure, could produce such large wave-

* *Comptes Rendus*, **65**, 292, 1867.
† *Ibid.*, **74**, 516, 1872.
‡ *Nature*, **23**, 604, 1881.
§ *Copernicus*, **3**, 206, 1884.
‖ *Astronomy and Astrophysics*, **13**, 448, 1894.
¶ *Lick Pub.*, **13**, Part 6, 1918.
** *Pub. D.A.O.*, **2**, No. 16, 1924.
†† *Ap. J.*, **52**, 39, 1920; *M.N.*, **81**, 142, 143, 1920.
‡‡ *H.B.*, Nos. 834, 836, 842, 843, 1926, 1927.



1 and 2, Wolf-Rayet Star B.D. 37° 3821; 3 and 4, Wolf-Rayet Star B.D. 35° 4013; 5 and 6, Nova Cygni (1920); 7 Nova Aquilæ (1918); 8 and 9, P. Cygni.

length displacements under the conditions believed to exist in stellar atmospheres. In the present paper some observational evidence in favour of a Doppler-effect explanation is put forward, and a mechanism for the production of the bands is suggested which is able to account for many of the observed peculiarities of Wolf-Rayet spectra. The paper is based on spectrograms taken by J. S. Plaskett and by the present writer.

Measurements of Band Width.

Quantitative evidence for the existence of Doppler effects may be derived from measures of band width. If, as has been suggested, the broadening is due to motions of the emitting gases in the line of sight, then the widths of bands due to the same atom in different regions of the spectrum should be proportional to wave-length. To test this point careful measures of the bands in a number of stars have been made, and the results of the measures are contained in Table I. In this table the band width has been denoted by $\Delta\lambda$, and a comparison has been made between the observed width and that calculated by means of a linear formula. The multiplying factor has been obtained by taking the average of the quantity, "$\Delta\lambda$ observed/λ" for the series of bands under consideration.

An inspection of the table shows that in every case there is an increase in band width with wave-length, though the rate of increase is somewhat greater than that predicted by a theory of Doppler broadening. In the first three stars in the table the Balmer and Pickering series have been measured. In these stars the discrepancy between observation and theory may well be due, at least in part, to diminishing contrast with the continuous spectrum in passing to higher series members. The two bands λ 4650 and λ 5695 have been measured in three of the stars. λ 4650 is known to be due to C^{++} and λ 5695 has been tentatively assigned to the same ion, though this identification is by no means certain. It is possible, therefore, that the discrepancies for these bands may be due to different atoms being involved. In any case, the bands show an increase in width with wave-length, and the evidence as a whole tends strongly to confirm the hypothesis that the broadening of the bands is due to Doppler effects. This hypothesis, then, serves as a starting-point in formulating a more complete theory of Wolf-Rayet emission.

Comparison with Novæ.

It is well known that the emission spectra of novæ consist of broad bands, strongly resembling those found in the spectra of Wolf-Rayet stars. This similarity in appearance naturally suggests that similar causes may be at work in the production of the spectra. For purposes of comparison, a spectrogram of Nova Aquilæ (1918) taken by Dr. J. S. Plaskett and two spectrograms of Nova Cygni (1920) secured by Mr. W. E. Harper have been reproduced in Plate 7. These spectra are part of an

TABLE I.

Widths of Wolf-Rayet Bands.

B.D. 37° 3821.

Balmer Series.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00833$.

λ.	Δλ Obs.	Δλ Calc.
6563	58·0	54·7
4861	41·2	40·5
4340	33·4	36·2

Pickering Series.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00782$.

λ.	Δλ Obs.	Δλ Calc.
5411	41·6	42·3
4541	35·5	35·5
4200	33·4	32·8

B.D. 35° 4001.

Balmer Series.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00883$.

λ.	Δλ Obs.	Δλ Calc.
6563	57·5	57·9
4861	43·9	42·9
4340	37·8	38·3

Pickering Series.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00961$.

λ.	Δλ Obs.	Δλ Calc.
5411	53·0	52·0
4541	40·8	43·6
4200	42·2	40·4

B.D. −23° 4553.

Balmer Series.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00961$.

λ.	Δλ Obs.	Δλ Calc.
6563	74·2	63·1
4861	44·8	46·7
4340	36·0	41·7

Pickering Series.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00892$.

λ.	Δλ Obs.	Δλ Calc.
5411	50·8	48·3
4541	40·5	40·5
4200	35·5	37·5

B.D. 43° 3571.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}01707$.

λ.	Δλ Obs.	Δλ Calc.
5695	102·3	97·2
4650	75·2	79·4

B.D. 35° 4013.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00711$.

λ.	Δλ Obs.	Δλ Calc.
5695	43·6	40·5
4650	30·5	33·6

B.D. 36° 3956.

$\Delta\lambda$ Calc. $= \lambda \times 0{\cdot}00863$.

λ.	Δλ Obs.	Δλ Calc.
5695	52·4	49·1
4650	37·5	40·1

excellent series of plates of Nova Aquilæ and Nova Cygni taken at this observatory, to which I have had access for the purpose of comparing the spectra of novæ and Wolf-Rayet stars. The result of this comparison has been to confirm the idea that the two types of spectra are fundamentally similar. This conclusion is based on the following points of resemblance.

The bands of both novæ and Wolf-Rayet stars are very strong relative to the continuous spectrum, and make a considerable contribution to the visual and photographic magnitude of these stars. This great concentration of energy in emission wave-lengths is found in no other type of purely stellar spectrum, and strongly suggests that the origin of the emission for the two types of objects is the same.

The wave-lengths of the centres of nova bands are approximately undisplaced from their normal positions. The same is true of Wolf-Rayet bands, as may be seen from the table of wave-lengths published by J. S. Plaskett * and from my own unpublished measures.

The widths of nova bands are of the same order as Wolf-Rayet bands, and the bands of both types of spectra show a progressive increase in width with wave-length. The measures of Wolf-Rayet bands have already been given in Table I. Measures of the bands of Nova Aquilæ and Nova Cygni in their later stages are shown in Table II. The bands of Nova Aquilæ are quite sharp, while those of Nova Cygni are exceedingly diffuse and difficult to measure. The measures for Nova Aquilæ are due to Lunt,† while Harper's ‡ measures have been given for Nova Cygni. Deviations from the values predicted from a theory of Doppler broadening are large for Nova Cygni and small for Nova Aquilæ. It is significant to note that deviations from the calculated values are of the same character for both novæ and Wolf-Rayet stars, and indicate a more rapid increase in width, in passing to the red region, than would be predicted by theory.

TABLE II.

Widths of Nova Bands.

Nova Aquilæ (1918) (*Lunt*). $\Delta\lambda$ Calc. $=\lambda\times 0{\cdot}01104$.			*Nova Cygni* (1920) (*Harper*). $\Delta\lambda$ Calc. $=\lambda\times 0{\cdot}00532$.		
λ.	Δλ Obs.	Δλ Calc.	λ.	Δλ Obs.	Δλ Calc.
5006·9	55·9	55·3	6563	39·9	34·9
4959·1	56·1	54·7	5169	30·9	27·5
4861·5	53·8	53·7	5018	25·0	26·7
4641·1	51·5	51·2	4924	27·0	26·2
4363·4	48·8	48·2	4861	28·0	25·9
4340·6	46·5	47·9	4341	22·6	23·1
4102·0	44·2	45·3	4102	18·8	21·8
			3933	17·8	20·9

One of the most significant points of resemblance between novæ and Wolf-Rayet stars is the appearance of absorption on the violet edges of emission bands. Such displaced absorption, which assumes somewhat different forms for different stars, is a conspicuous feature

* *Pub. D.A.O.*, **2**, No. 16, 1924.
† *M.N.*, **80**, 523, 1920.
‡ *Pub. D.A.O.*, **1**, 270, 1921.

of the spectra of novæ. Absorption on the violet edges of Wolf-Rayet bands has been observed by Wright,* Plaskett,† Perrine,‡ Miss Payne,§ and by the present writer. Perrine has observed absorption lines to the violet of the bands of γ Argus which vary both in intensity and position. He concluded that they are due to motion of the absorbing gases, and that they suggest an analogy with the novæ. The phenomenon is less conspicuous in Wolf-Rayet stars than in novæ, but of its existence there can be little doubt. The strength of the absorption appears to be different for different bands, and is strongest for the bands of *He* I and *C* III. In Plate 8 are reproduced microphotometer tracings of a number of Wolf-Rayet spectra secured at this observatory. The tracings clearly show absorption on the violet edges of the wave-lengths 4650 *C* III, 4471 *He* I, and 3889 *He* I.

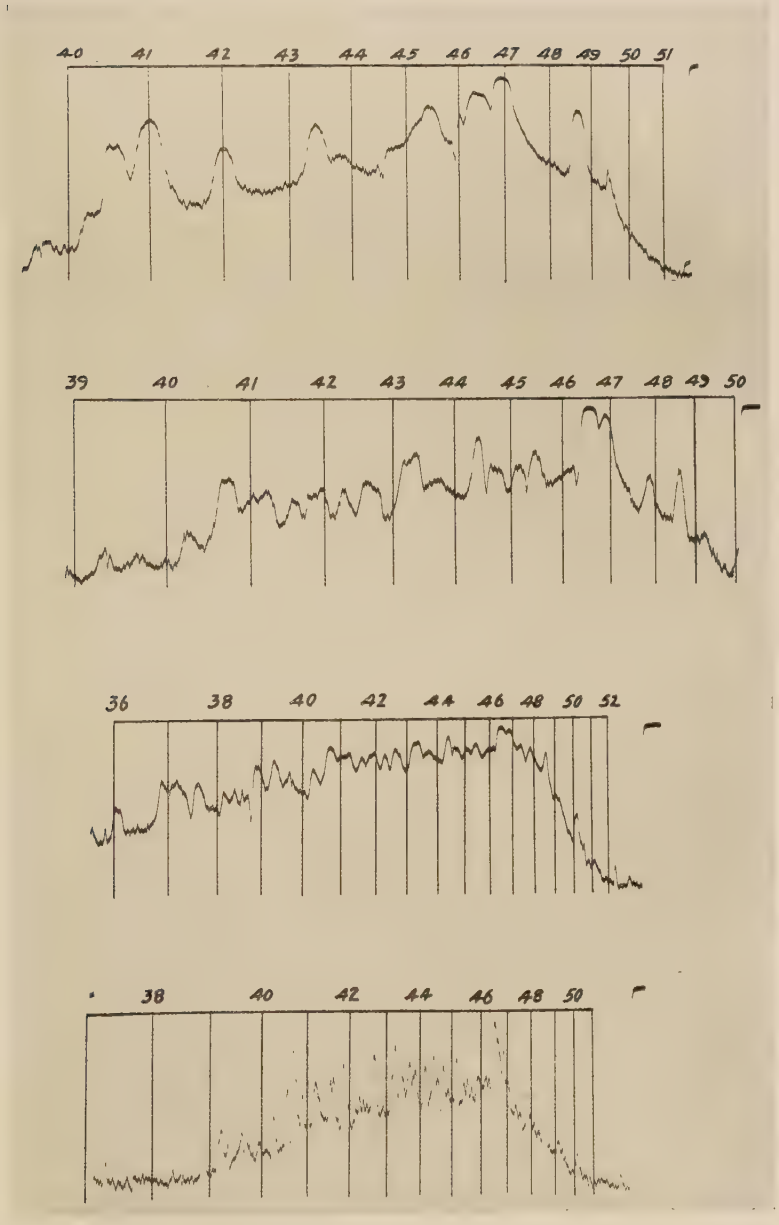
Proposed Theory of Wolf-Rayet Emission.

The similarities with the spectra of novæ, outlined above, have suggested that the emission spectra of Wolf-Rayet stars may be due to a cause similar to that which is generally considered to be responsible for the band spectra of novæ—namely, the continuous ejection of gaseous material from the star. Practically all the characteristic features of Wolf-Rayet spectra may be explained on the assumption that gaseous material is continuously being ejected in a radial direction and with high velocity into space. On this theory the star is surrounded by an expanding envelope of nebulous gases, whose extension in space is large, relative to the diameter of the star itself. As in novæ,‖ the gases on the side of the star nearer to the observer will, owing to their velocity, be transparent to radiation from other parts of the nebulous envelope, all parts of which will contribute to the observed intensity of the emission bands.

Both the width of the bands and the absorption on their violet edges have a logical explanation on this hypothesis. The width of the bands will depend on the maximum positive and negative velocities in the line of sight. The filling in of the band between the two extremes may be attributed to the spread in velocities and to the fact that the velocities of the radiating gases make angles with the line of sight having all values from 0° to 180°. The dark borders on the violet edges of bands may be explained as due to absorption of the continuous spectrum of the star by that part of the nebulous envelope between the star and the observer. Since the gases in this part have the largest negative velocity in the line of sight, the position of the absorption would be expected to coincide with the violet edge of an emission band.

The great intensity of emission wave-lengths relative to the continuous spectrum in the stellar region of observation may be explained on the assumption that the light of the star is relatively rich in light of very short wave-lengths. This light is absorbed by the nebulous

* *Lick Pub.*, **13**, 224, 1918.
† *Pub. D.A.O.*, **2**, No. 16, 1924.
‡ *Ap. J.*, **52**, 39, 1920.
§ *H.B.*, No. 844, 1927.
‖ See Dingle, *Modern Astrophysics*, p. 272, 1924.



Tracings of Wolf-Rayet Spectra.

envelope and re-emitted in the longer wave-length regions. Such a mechanism has obvious similarities to that suggested by Hubble * and Zanstra † to explain the spectra of planetary nebulæ, and by Rosseland ‡ to account for ordinary stellar emission. In this connection it is of interest to note that Wolf-Rayet spectra show a remarkable similarity to the spectra of planetary nebulæ in the comparative distribution of energy between the continuous spectrum (due to the star) and the emission wave-lengths (due to the nebulosity) in the stellar region of observation.

Old Novæ as Wolf-Rayet Stars.

The fact that old novæ have, in some cases, been shown to be Wolf-Rayet stars, lends a certain amount of support to the theory of the origin of Wolf-Rayet emission that has just been outlined. Adams and Pease § have shown that Nova Aurigæ (1891) and Nova Persei (1901), twenty-three years and fourteen years respectively after the initial outbursts, were typical Wolf-Rayet stars in so far as the observations, made with small dispersion apparatus, could determine. Since there seems no reason for supposing that the agency responsible for the broad band emission in the final state of a nova is different from that causing the bands in its earlier stages, these observations appear to indicate that the cause of nova and Wolf-Rayet emission is the same.

Ejection of Atoms by Selective Radiation Pressure.

The conclusion that Wolf-Rayet emission is caused by continuous ejection of gaseous material from a star is a deduction from the observations which is not necessarily dependent on any specific assumption as to the forces responsible for such ejection. It is obvious, however, that the validity of the theory depends on the existence of some agency which is at once capable of producing the observed effects and likely to be operative under stellar conditions. Such an agency may be found in the force exerted on atoms by selective radiation-pressure in the manner discussed by Milne and others in connection with theories of the solar chromosphere. In one of his earlier papers on the theory of the solar chromosphere, Milne || suggests the possibility of the continuous ejection of high-speed atoms from stars by radiation-pressure. In a later paper ¶ the mechanism responsible for the ejection of atoms is developed in sufficient detail to permit of a calculation of the limiting velocities likely to be acquired. These turn out to be of the order of several hundred km. per sec., a figure which approximates to the order of the hypothetical velocity effects observed in novæ and Wolf-Rayet stars, which vary from 300 km. for stars with narrow bands to 2400 km. for B.D. 43° 3571. A similar mechanism is considered by Milne to be responsible for the emission bands and displaced absorption lines of novæ.

* *Ap. J.*, **56**, 162, 1922.
† *Ibid.*, **65**, 50, 1927.
‡ *Ibid.*, **63**, 226, 1926.
§ *Ibid.*, **40**, 294, 1914.
|| *M.N.*, **84**,354, 1924.
¶ *Ibid.*, **86**, 459, 1926.

M. C. Johnson,* investigating the same subject, has put forward a suggestion which may be of considerable value for the interpretation of certain anomalous line intensities in Wolf-Rayet spectra. His work appears to show that the value of R/G, the ratio of radiation-pressure to gravity at the surface of a star, may have widely different values for different atoms and for atoms in different stages of ionization. Thus, for a giant star at a temperature of 30,000° the values of R/G for *H*, *He*, and He^+ have been calculated as follows :—

Atom.	R/G.
H	$40/3\times10^3$
He	$190/3\times10^3$
He^+	$2{\cdot}5\times10^5/3\times10^3$

Other ions likely to have high values of R/G are C^{++}, Si^{+++}, and probably O^{++} and N^{++}.

From this it appears that the intensity of a given emission band may depend partly on a selective effect due to radiation-pressure, since it seems reasonable to suppose that atoms for which R/G is largest would be given off most abundantly. An effect such as this is undoubtedly present in the sun, causing the appearance of Ca^+, almost alone, in the highest levels of the chromosphere. In Wolf-Rayet stars hydrogen appears to be abnormally weak (the bands in the approximate positions of the Balmer lines are probably mainly due to He^+), while *He* II and *C* III appear with much greater intensity than would be expected from their behaviour in absorption O-type spectra. The probability that these anomalous intensities are due to the selective effect of radiation-pressure removes a serious difficulty, and points to the necessity for revising some previously held ideas concerning the classification of these stars.

Loss of Mass due to Ejection of Stellar Material.

In connection with the theory that Wolf-Rayet emission is a result of the ejection of stellar material, it is of some importance to calculate the loss of mass which a star may sustain due to such ejection. In order for the observed frequency of occurrence of Wolf-Rayet stars to be accounted for, the duration of the Wolf-Rayet stage must be considerable even on the assumption that every star in the sky becomes a Wolf-Rayet star at some time in its history. The loss of mass has been calculated, first for a star of mass and radius equal to those of the sun, and secondly for a star of 30 times the sun's mass and radius equal to 100,000,000 km. The velocity of ejection has been taken as equal to 2000 km. per second, greater than necessary to account for the width of most Wolf-Rayet bands. The density is that given by Milne † for the solar chromosphere, 3×10^{-17} gm. per c.c. This value is probably rather high, even for the solar chromosphere, but it provides

* *M.N.*, **86**, 300, 1926.
† *Ibid.*, **85**, 141, 1924.

an upper limit for the rate of diminution of the star's mass. The result of the calculation is as follows :—

Star with mass and radius of sun :
Fraction of mass lost per year $= 6 \times 10^{-12}$.

Star with mass $30 \times \odot$ and radius 100,000,000 km. :
Fraction of mass lost per year $= 4 \times 10^{-9}$.

Since these are upper limits, it would appear from the above calculation that the rate of diminution of mass is not sufficiently large to offer a serious obstacle to the theory.

P Cygni, η Carinæ, and Stars of Similar Type.

The spectrum of P Cygni consists of strong, moderately narrow, emission lines, bordered on their violet edges by strong absorption. The displaced absorption lines correspond to a velocity of 82 km./sec. The small displacement of the centres of emission lines from their normal positions is believed to be due to the radial velocity of the star. Measures of the widths of emission lines by Frost * and by the present writer indicate that for lines due to a given atom the width increases with increasing wave-length. Means of my own measures of the lines of hydrogen and helium are given in Table III. The increase in width of these lines is more rapid than a simple linear formula connected with wave-length would indicate, as may be seen from an inspection of the table. In this the behaviour of the lines is similar to that of the lines of Nova Cygni (1920), measures of which have already been given. The bands of Nova Geminorum (1912) also, immediately after maximum, have been shown by Wright † to vary as the square of the corresponding wave-length and not directly as the wave-length. Since the width of the nova bands is generally attributed to motions of the emitting gases, it would appear that the Doppler effect is not necessarily ruled out as a cause for the widths of P Cygni lines. Rapidly decreasing intensities in passing to higher series members, combined with errors of measurement caused by the varying dispersion of the spectrograph, may account for the observed differences.

The star η Carinæ also has an emission spectrum, and the emission lines have been observed by Lunt, ‡ Miss Cannon, § and Perrine to be bordered on their violet edges by absorption. There appears to be no information available as to the variation of the width of emission lines with wave-length. Both P Cygni and η Carinæ have been numbered among the novæ by Stratton, ‖ though the course of their variation has differed somewhat from that followed by most novæ. This similarity with novæ, considered in connection with the absorption on the violet edges of emission lines and the variation in width of P Cygni lines with wave-length, suggests that the peculiarities in the spectra of these

* *Ap. J.*, **35**, 286, 1912.
† *Lick Pub.*, **14**, 36, 1926.
‡ *M.N.*, **79**, 621, 1919.
§ *H.A.*, **28**, 175, 1901.
‖ *Handbuch der Astrophysik*, Band 6, p. 286, 1928.

stars is due to the ejection of gaseous material in a manner similar to that suggested for Wolf-Rayet stars.

Table III.

Widths of P Cygni Emission Lines.

He I $\Delta\lambda$ Calc. $=\lambda\times 0{\cdot}000803$.			*H* $\Delta\lambda$ Calc. $=\lambda\times 0{\cdot}001040$.		
λ.	$\Delta\lambda$ Obs.	$\Delta\lambda$ Calc.	λ.	$\Delta\lambda$ Obs.	$\Delta\lambda$ Calc.
5875·6	5·5	4·7	6562·8	8·7	6·8
4471·5	3·0	3·6	4861·3	4·8	5·1
			4340·5	4·0	4·5
			4101·7	3·8	4·3

Of somewhat similar character are the two stars H.D. 45910 and B.D. 11° 4673, whose spectra have been described by J. S. Plaskett * and by Merrill † respectively. Both of these stars have emission lines with dark borders on their more refrangible edges. They have shown, in addition, certain peculiar variations in their absorption spectra which serve to indicate a relationship with the novæ. These stars are individual members of a considerable class of stars which show the P Cygni characteristic of emission lines with dark borders on their violet edges. These stars may be assumed to have certain physical characteristics in common, though the precise explanation of their peculiarities is still a matter for speculation. There seems some reason to suppose that the puzzle presented by their peculiar emission may be solved, in part at least, by the application of the present theory of emission to their spectra.

Summary and Discussion.

As a result of the observation and measurement of Wolf-Rayet spectra a theory of stellar emission has been put forward which postulates the continuous ejection of high-speed atoms from a star by selective radiation-pressure. The observational data have been shown to be in agreement with this theory, and it has been found possible to provide a logical explanation of the width of Wolf-Rayet bands, their variation in width with wave-length, the appearance of absorption on their violet edges, and the great intensity of the bands relative to the continuous spectrum. A calculation of the rate of loss of mass of a star by this mechanism appears to indicate that it is not sufficiently rapid to cause doubts as to the validity of the theory. Finally, there appears good reason to believe that the spectral peculiarities of P Cygni, η Carinæ, and other stars which show the P Cygni characteristic may be explained, in part at least, on the basis of the same theory.

The relationship which appears to exist between Wolf-Rayet stars

* *Pub. D.A.O.*, **4**, No. 1, 1927. † *Ap. J.*, **69**, 330, 1929.

and stars of P Cygni type naturally raises the question as to the physical causes operating in the two cases to produce spectra which in some respects are widely different and in others appear to be fundamentally similar. In view of the uncertainty which exists concerning the nature of these bodies it is desirable to proceed with caution in attempting any explanation. There are, however, certain elementary theoretical considerations which suggest themselves and which may lead to a clearer understanding of the problem.

The possibility of the ejection of atoms from a star by radiation-pressure depends on two factors: (*a*) the temperature of the photosphere, and (*b*) the value of gravity at the stellar surface. The work of Johnson * has shown that the probability of atoms of a given sort being ejected is by no means a simple function of these two variables, even when certain perhaps unjustifiable assumptions are made concerning the depth of the absorption lines. Nevertheless it would appear, in general, that such ejection would be rendered more probable or increased in amount by increases in temperature or by decreases in the value of surface gravity. Although there are not at present any published measures of the distribution of intensity in the continuous spectra of Wolf-Rayet stars, the high degree of ionization in their atmospheres, the great intensity of Wolf-Rayet bands relative to the continuous spectrum, and the occurrence of Wolf-Rayet stars as the nuclei of planetary nebulæ, all point to very high temperatures in the photospheres of these stars. P Cygni stars, on the other hand, are generally assumed to be of lower temperature. From what has been said concerning the effect of high temperature and low surface gravity, combined with the knowledge we have of other early type stars, it seems logical to suppose that Wolf-Rayet stars are very massive, diffuse, and highly luminous objects, differing from ordinary absorption O-type stars mainly in their lower density and lower values of surface gravity, though perhaps possessing higher photospheric temperatures as well. The same may probably be said of stars of P Cygni type as compared with absorption-line stars of the same spectral class.

As regards a comparison between the two types of objects, the process of reasoning outlined above would suggest that, since the Wolf-Rayet stars have the higher photospheric temperatures, the P Cygni stars are characterized by lower density and lower values of surface gravity. If, then, we are correct in designating the Wolf-Rayet stars as giants, we should, perhaps, think of the P Cygni stars as super-giants.

The ideas presented in this paper were developed during an extended investigation of Wolf-Rayet stars which included all the northern objects of this type down to magnitude 11·0. A complete account of this investigation, including wave-length data as well as a discussion of the classification of these stars, will appear in the publications of this observatory. The work was undertaken at the suggestion of the director, Dr. J. S. Plaskett, whose helpful advice and kind interest throughout the investigation are very gratefully acknowledged. It is a

* *M.N.*, **86**, 300, 1926.

pleasure also to acknowledge assistance received from other members of the staff. In particular, I am indebted to Mr. W. E. Harper for a discussion of the data on novæ, to Mr. R. O. Redman for criticism and suggestions on the theoretical aspects of the investigation, and to Mr. J. A. Pearce for the loan of plates of P Cygni. I wish at the same time to thank Professor A. Pannekoek for the use of a three-prism plate of P Cygni, and Dr. A. V. Douglas, who prepared the microphotometer tracings which have been reproduced.

Dominion Astrophysical Observatory,
Victoria, B.C., Canada :
1929 *September* 23.

The Distribution of the Double Stars along the Giant Series.

By Dr. E. A. Kreiken.

In several earlier papers * various reasons were discussed why the double stars should statistically be considered to be dwarfs. There can, however, be no doubt that giant visual doubles and spectroscopic ones do occur, though I should expect their relative frequency to be far less among giants than among dwarfs. Moreover, both with the visual and with the spectroscopic doubles a selected material was used, and therefore a direct physical interpretation of our results might lead to erroneous conclusions. With the visual doubles our results were based on the pairs with $d \leqslant 15''$ and $\Delta m \leqslant 2{\cdot}5$, which are contained in the Southern Double Star Catalogue. As a rule we may expect the apparent angular distance d to be larger with the dwarfs than with the giants. Therefore it is not probable that many giant doubles have been excluded. On the other hand, the numbers of double stars with $d < 1''$ may statistically be incomplete. So with the giants of fainter apparent magnitude a number of double stars might have escaped observation. If this were the case the numbers of faint double stars would increase. Now our conclusions were partly based on the fact that the increase of the double star numbers with the fainter magnitudes is much larger than the corresponding increase of the stars in general. If therefore the numbers of double stars were influenced by an effect of selection, this would only strengthen our conclusions.

With the spectroscopic doubles we used those systems for which the spectra of both the main star and its fainter companion have been observed. Evidently the spectrum of the companion will be most easily observed if the mass-ratio between the two components is small. According to Vogt,† this mainly occurs with the stars of the main sequence. So here we may expect selection to have influenced our results. G. Shajn has treated the eclipsing variables and has found that they are mainly dwarfs. Evidently the variability of such a system will be most easily discovered if the luminosities of the two

* *B.A.N.*, **173**, 1929 ; *M.N.R.A.S.*, April 1929 ; *M.N.R.A.S.*, May 1929.
† *Zeitschrift für Physik*, **26**, 139, 1924.

components are nearly equal. So here also a selection of stars belonging to the main sequence may occur.*

In order to obtain definite conclusions I have considered the possibility of determining the relative frequency of double stars, both the visual and spectroscopic ones, among the different spectral types of the giant series and the main sequence. For this we want material for which our knowledge as to the variability of radial velocity and the occurrence of visual double stars is as complete as possible. I therefore considered in the first place the stars brighter than visual magnitude 5·51. For all these stars the radial velocities have been determined at the Lick Observatory. The stars contained in the Lick Radial Velocity Catalogue are of course largely giants, and therefore I have limited myself in the present paper to the investigation of the relative frequencies of the double systems along the giant series and the earlier types of the main sequence. As spectroscopic doubles all those pairs have been considered for which

(*a*) The velocity of the centre of gravity is given. In the Lick Catalogue these pairs are indicated by the symbol V.S.

(*b*) A provisional value of the velocity of the centre of gravity is given, *i.e.* those stars which in the Lick Catalogue are indicated by P.

(*c*) The radial velocity was found to be variable.

Doubtful cases indicated by " variable ? " have also been included.

The Lick Catalogue gives an appendix containing a list of stars for which no radial velocity measures were made at the Lick Observatory, but which, at other observatories, were discovered to have a variable radial velocity. These of course were also included.

For a small number of spectroscopic doubles the variability of the radial velocity must be ascribed to internal pulsation. It seems not quite justifiable to exclude these stars, as according to theory double stars may originate from the variables on the giant series.†

Therefore two solutions were made, one in which they were excluded, and one in which they were included.

The visual double stars were taken from Burnham's General Catalogue and from the Southern Double Star Catalogue. Both catalogues contain a large number of pairs for which it seems doubtful whether they really are physically connected. Especially with the wider pairs with large magnitude difference many optical pairs will occur. So it is necessary to fix certain limits outside of which the double stars will be considered as being optical ones only.

According to Struve, the number of optical pairs with a mutual distance $\leqslant d$ will be equal to

$$D = \frac{\pi d^2}{O} \times \frac{n(n-1)}{2},$$

where n is the total number of stars counted on the surface O. In

* Comp. Beer, *Veröff. Berlin-Babelsberg*, Band V., Heft 6 (1927), and Å. Wallenquist, *Publ. Bosscha Obs. Misc. Papers*, No. 1, 1929.

† Jeans, *M.N.*, **85**, 797, 1925 ; *Ten Bruggencate, Naturw.*, **14**, 905, 1926.

the *B.A.N.*, **151**, 1928, I have given a modified form of this formula, viz.:

$$D_0(m) = \frac{d^2}{r^2} A(m) \left\{ \frac{A(m) - 1}{2} + N(m_1) - N(m) \right\},$$

where $A(m)$ is the number of stars between the magnitude limits $m - \frac{1}{2}$ and $m + \frac{1}{2}$, and $N(m)$ the total number of stars up to the limiting magnitude m. We want the number of optical pairs with apparent magnitude m and with magnitude difference $\leqslant \Delta m$ (m being the magnitude of the brighter star), and therefore we write

$$D_0(m_1 \Delta m) = \frac{d^2}{r^2} A(m) \left\{ \frac{A(m) - 1}{2} + N(m + \Delta m) - N(m) \right\}.$$

For the number of optical pairs of the apparent magnitude m and magnitude difference between the limits Δm_1 and $A m_2$ we have:

$$D_0\left(m_1 \begin{matrix} \Delta m_1 \\ \Delta m_2 \end{matrix} \right) = \frac{d^2}{r^2} A(m) \{ N(m + \Delta m_2) - N(m + \Delta m_1) \} \quad . \qquad (1)$$

For m we adopted the limits 3·0, 4·0, and 5·0. Then the total numbers of optical pairs over the whole sky are as given in Table I. The values

TABLE I.

Computed Numbers of Optical Pairs.

(*a*) $m = 3{\cdot}0$.

d. / Δm.	<1″.	1″–2″.	2″–5″.	5″–10″.	10″–15″.	15″–30″.	30″–60″.	60″–120″.
0·0								
	0	0	0	0	0	0	1	3
1·0								
	0	0	0	0	0	0	1	3
2·0								
	0	0	0	0	0	1	3	12
3·0								
	0	0	0	0	1	2	7	30
4·0								
	0	0	0	1	1	5	22	87
5·0								
	0	0	1	1	3	15	61	243
6·0								
	0	0	2	4	8	40	164	654
7·0								
	0	1	2	13	19	106	424	1695
8·0								
	0	2	8	30	49	268	1072	4287
9·0								
	1	3	20	71	119	641	2565	10260
10·0								
	2	7	46	165	275	1485	5940	23760
11·0								
	5	14	100	359	598	3226	12906	51624
12·0								
	10	30	210	749	1249	6743	26973	107892
13·0								
	19	78	386	1449	2391	13047	52110	208440
14·0								
	36	107	749	2675	6670	21893	96390	385560
15·0								

TABLE I.—*continued.*

(*b*) $m = 4{\cdot}0$.

Δm. \ d.	<1″.	1″–2″.	2″–5″.	5″–10″.	10″–15″.	15″–30″.	30″–60″.	60″–120″.
0·0	0	0	0	0	0	1	2	9
1·0	0	0	0	0	0	2	5	21
2·0	0	0	0	1	1	3	17	66
3·0	0	0	0	2	2	11	46	183
4·0	0	0	1	4	5	31	125	498
5·0	0	0	3	9	16	84	334	1338
6·0	0	1	7	24	41	219	874	3498
7·0	1	2	17	62	102	550	2204	8814
8·0	2	6	40	146	243	1309	5238	20952
9·0	5	13	95	339	565	3051	12204	48816
10·0	10	30	206	737	1229	6635	26541	106164
11·0	21	61	431	1539	2561	13837	55350	221400
12·0	40	120	832	2975	4965	26708	107280	428760
13·0	73	292	1465	5490	9150	49410	197640	790560
14·0								

(*c*) $m = 5{\cdot}0$.

0·0	0	0	0	1	1	5	21	84
1·0	0	0	0	2	2	12	48	192
2·0	0	0	1	4	6	34	136	543
3·0	0	1	2	11	17	95	378	1512
4·0	0	1	8	29	76	255	1018	4071
5·0	1	3	20	73	121	659	2633	10530
6·0	2	8	51	185	305	1663	6642	26568
7·0	6	18	122	440	727	3961	15822	63288
8·0	14	40	286	1020	1686	9194	36720	146880
9·0	30	90	620	2220	3670	20010	79920	317680
10·0	62	186	1297	4635	7663	41777	166860	667440
11·0	119	361	2317	9113	14756	80444	321300	1385200
12·0	221	874	4427	16578	27404	149396	596700	2386800
13·0								

of N(m) necessary for the computations were taken from the tables by Seares, van Rhijn, Joyner, and Richmond.* We note the enormous increase of the numbers of optical pairs with increasing values of d and Δm. If the limits of d and Δm were chosen sufficiently large every bright star would be a double or multiple system. It is therefore necessary to fix the limits of d and Δm in such a way that only a relatively small number of optical pairs is included in our investigations. In Table II. the observed numbers of double stars have been given. Within the limits indicated by thick broken lines the observed numbers are larger than the computed ones, and therefore it is probable that most of the pairs within these limits really are physically connected. The limits adopted here are rather severe, and there can be no doubt that some real binary systems will be excluded. Sirius and its dwarf companion fall outside the limits adopted here. Nevertheless if wider limits were adopted the percentage of optical pairs among the double stars would be too large.

TABLE II.

Observed Distribution of Double Stars.

(a) $m < 3{\cdot}5$.

Δm. \ d.	<1″.	1″–2″.	2″–5″.	5″–10″.	10″–15″.	15″–30″.	30″–60″.	60″–120″.	>120″.
0·0									
	4	2	2	1	0	0	0	0	0
1·0									
	2	0	1	0	1	0	0	0	1
2·0									
	0	0	1	1	1	1	2	0	1
3·0									
	2	0	3	0	0	0	1	1	0
4·0									
	0	2	2	0	1	3	2	0	0
5·0									
	1	0	2	1	0	3	1	1	2
6·0									
	0	0	1	2	1	0	0	1	2
7·0									
	0	0	2	0	1	0	0	1	3
8·0									
	0	0	3	0	2	2	3	1	2
9·0									
	1	0	1	0	1	2	5	2	0
10·0									
	0	0	1	2	0	3	1	3	0
11·0									
	0	0	0	1	0	5	4	0	0
12·0									
	0	0	0	1	0	0	2	0	0
13·0									
	0	0	0	0	0	1	1	1	0
14·0									
	0	0	0	0	0	0	1	0	0
15·0									

* *Astrophys. Journal*, **62**, 1925.

TABLE II.—*continued.*

(*b*) $m = 3{\cdot}5–4{\cdot}5$.

Δm. \ d.	<1″.	1″–2″.	2″–5″.	5″–10″.	10″–15″.	15″–30″.	30″–60″.	60″–120″.	>120″.
0·0	8	6	4	0	1	2	0	0	1
1·0	9	3	4	0	1	0	2	1	0
2·0	4	2	2	0	1	2	3	7	1
3·0	1	1	4	4	0	1	5	2	2
4·0	1	2	1	3	0	2	2	3	1
5·0	0	0	1	3	0	5	7	2	0
6·0	0	2	0	2	1	6	5	1	1
7·0	0	0	2	4	2	3	6	4	0
8·0	0	0	1	1	0	1	7	1	0
9·0	0	0	1	2	0	7	3	1	0
10·0	0	0	0	0	0	2	1	0	0
11·0	0	0	0	0	0	0	1	0	0
12·0	0	0	0	0	1	0	0	0	0
13·0	0	0	0	1	0	0	1	0	0
14·0									

(*c*) $m = 4{\cdot}5–5{\cdot}5$.

0·0	33	8	6	9	1	5	3	4	4
1·0	12	6	7	12	3	2	7	4	4
2·0	2	5	9	7	6	7	6	6	2
3·0	3	9	9	4	0	6	11	6	4
4·0	1	4	11	5	4	6	9	5	3
5·0	1	4	12	7	9	9	11	4	0
6·0	1	4	8	4	4	17	16	0	2
7·0	0	0	1	9	2	8	6	2	0
8·0	0	1	4	6	4	13	4	2	0
9·0	0	0	3	1	2	3	1	1	0
10·0	0	0	0	0	0	2	1	0	0
11·0	0	0	0	1	0	2	2	0	0
12·0	0	0	1	1	0	1	1	0	0
13·0									

For a very large number of stars contained in the Lick Radial Velocity Catalogue spectroscopic parallaxes are available. They were taken from the lists published by the Mount Wilson observers, those from Victoria, or from the one given by Edwards. A considerable number of trigonometric parallaxes were taken from Schlesinger's Catalogue of Trigonometric Parallaxes (1924), and additional ones were obtained from newer sources.

For a third group of stars parallaxes derived from group motion were available.*

The remaining stars were treated in the following way :—

(*a*) For the A, F, G, K, and M stars the total proper motion was taken from the Boss Catalogue. The mean proper motion parallaxes were then obtained by interpolation in the tables by van Rhijn. †

(*b*) For the B type stars the mean absolute magnitudes as derived by Adams and Joy ‡ were used, viz. :

B0.	B1.	B2.	B3.	B4.	B5.	B6.	B7.	B8.	B9.
−3·1	−2·5	−1·6	−1·2	−0·9	−0·8	−0·6	−0·4	−0·2	0·7

(*c*) For the long period M variables the absolute magnitudes as derived by Merrill and Strömberg § were used, viz. :

M2*e*.	M3*e*.	M4*e*.	M5*e*.	M6*e*.	M7*e*.	M8*e*.
−0·6	−0·6	−0·5	0·0	0·5	0·6	0·6

(*d*) For the Cepheid variables the absolute magnitudes as derived by Shapley's period-luminosity curve were used.

In Table III. a survey of the available parallax material is given. If for a star both the spectroscopic and the trigonometric parallaxes were available, the spectroscopic one was used. In those cases the trigonometric parallax was not included in the counts of Table III.

This table gives for each separate spectral type in the first row the number of spectroscopic parallaxes ; in the second row the number of parallaxes obtained from trigonometric data, or from group motion, while the last row gives the numbers of parallaxes based on absolute magnitude or total proper motion.

Table III.

Numbers of Parallaxes for Different Spectral Types.

Spectrum.	O–B_7.	B_8–A_5.	F.	G.	K.	M.
Spectrosc. π	146	364	189	134	356	94
Trigon. π	173	110	28	22	41	15
$\pi m, \mu$	114	293	125	94	332	38

* J. C. Kapteyn, "On the Motion of Galactic Helium Stars," *Mount Wilson Contr.*, Nos. 82 (1914) and 147 (1918) ; N. H. Rasmuson, "A Research on Moving Clusters," *Lund Medd.*, Serie II., Bd. 3, Nr. 26, 1921.

† *Gron. Publ.*, **34**, Tables 37–38, 1923.

‡ "A Spectroscopic Method of deriving the Parallaxes of B-type Stars," *Mount Wilson Contr.*, No. 262, 1923.

§ *Mount Wilson Contr.*, No. 327, 1926.

TABLE IV.

Percentages of the Observed Numbers of Stars of different Spectral Types absolutely brighter than M = + 3·0 *and* + 2·0 *respectively.*

Spectrum.	O–B_7.	B_8–A_5.	F.	G.	K.	M.	All Stars.
Percentage M = +3·0	100	99	74	78	97	100	93
Percentage M = +2·0	98	89	47	69	90	100	84

TABLE V.

Distribution of Double Stars for M ≤ + 2·0.

	Numbers.						Percentages.					
	O–B_7.	B_8–A_5.	F.	G.	K.	M.	O–B_7.	B_8–A_5.	F.	G.	K.	M.
Totals	426	686	160	172	653	147	..	..	..	..	..	..
Spectroscopic d.	184	231	53	41	108	12	43	34	33	24	17	8
Spectroscopic d.	184	231	60	47	114	29	43	34	37	27	17	20
Visual d.	36	70	15	14	21	1	8	10	9	8	3	0·7
Visual d.	48	104	21	19	36	8	11	15	13	11	6	5
Normal	215	373	89	109	507	111	50	54	56	63	78	76
Multiple	22	22	10	3	3	1	5	3	4	3	0·6	0·7

TABLE VI.

Distribution of Double Stars for M ≤ + 3·0.

	Numbers.						Percentages.					
	O–B_7.	B_8–A_5.	F.	G.	K.	M.	O–B_7.	B_8–A_5.	F.	G.	K.	M.
Totals	432	760	252	196	707	147	..	..	..	..	..	..
Spectroscopic d.	187	262	74	50	113	12	43	34	29	25	16	8
Spectroscopic d.	187	262	81	56	119	29	43	34	32	29	17	20
Visual d.	37	75	26	20	21	1	9	10	10	10	3	0·7
Visual d.	49	115	37	25	37	8	11	15	15	13	5	5
Normal	218	409	145	120	560	111	50	54	58	61	79	76
Multiple	23	26	11	5	4	1	5	3	4	3	0·6	0·7

TABLE VII.

Mean Absolute Magnitude of the Observed Stars.

Spectrum.		O–B_7.	B_8–A_5.	F.	G.	K.	M.
$\overline{M}_{abs}$	(limit +2·0)	−1·09	+0·55	+0·52	+0·23	+0·43	−0·15
	(limit +3·0)	−1·04	+0·74	+1·26	+0·52	+0·59	−0·15

With the parallaxes obtained in the way described before, the apparent magnitudes were converted into absolute ones by means of the formula

$$M = m + 5 + 5 \log \pi.$$

Then the distribution of the stars by spectral type and absolute magnitude was investigated. As the material chiefly consists of absolutely very bright stars we limited our investigation to the stars with $m \leqslant + 3{\cdot}0$.

It appeared, however, that our results were not influenced if another limit, viz. $m \leqslant +2{\cdot}0$, was adopted.

From the numbers in Table IV. it appears that for the different spectral types the percentages of the total number of stars which are rejected by adopting these limits are small.

It is evident that by considering only the stars absolutely brighter than mag. + 3·0, only the giants will be taken into account with the M, K, and G stars. With the F stars a number of dwarfs will probably be included. With the A stars both branches of the luminosity curve fall within the range here considered. The early B stars and the O stars, though very bright, are dwarfs in this respect, that they belong to the main series. So evidently we can only consider here the giant series and the early types of the main sequence.

The Tables V. and VI. are drawn up with the limits $M_{abs} = +2{\cdot}0$ and + 3·0 respectively. In these tables I have given

(*a*) The total numbers of stars of each spectral type with absolute magnitude $< +2{\cdot}0$ and + 3·0 respectively.

(*b*) The numbers of stars of each separate spectral type having a variable radial velocity (see above) or a composite spectrum. Two series of values have been given. With the first one, stars with a variable magnitude have been excluded; with the second they are included. Algol variables have always been included.

(*c*) The numbers of visual doubles with $\Delta m < 4{\cdot}0$ and d between the limits $0''{\cdot}50$ and $15''$.

(*d*) The numbers of visual doubles counted within the limits of Table II.

(*e*) The numbers of stars which are not double. It should be remarked that these numbers are probably somewhat too large, as a number of visual binaries will fall outside the limits of Table II. Owing to the fact that there are a number of stars which are spectroscopic doubles and at the same time visual doubles, the combined numbers of visual doubles, spectroscopic ones, and single stars will not be equal to the total numbers of the stars of each spectral type.

(*f*) The numbers of multiple systems, *i.e.* the visual multiple systems and those which are spectroscopic as well as visual doubles.

In the second part of these tables the numbers of each separate group are expressed in percentages of the total numbers of stars.

From these tables it is very apparent that the percentage of stars which are double is greatest among the earlier spectral types. This is the case with the visual doubles and with the spectroscopic ones. The increase of the spectroscopic doubles with the A and B stars may partly be an effect of mass. The amplitude K of variation of the velocity curve of the spectroscopic doubles is proportional to

$$(1 - e^2)^{\frac{1}{2}} \cdot P^{-1} \cdot M\left(\frac{\gamma}{1+\gamma}\right)^3 \quad . \quad . \quad . \quad (2)$$

(e = eccentricity, P = period, M = mass, γ = mass-ratio.)

Evidently K increases with the larger masses, and consequently the variation of the radial velocity will be easier to detect. In Table VII. I have given the mean absolute magnitude for each separate spectral type for the stars used in the present investigation. As might have been anticipated, the mean absolute magnitude of the O and B stars is very low. For the other spectral types the mean absolute magnitude is pretty well a constant.

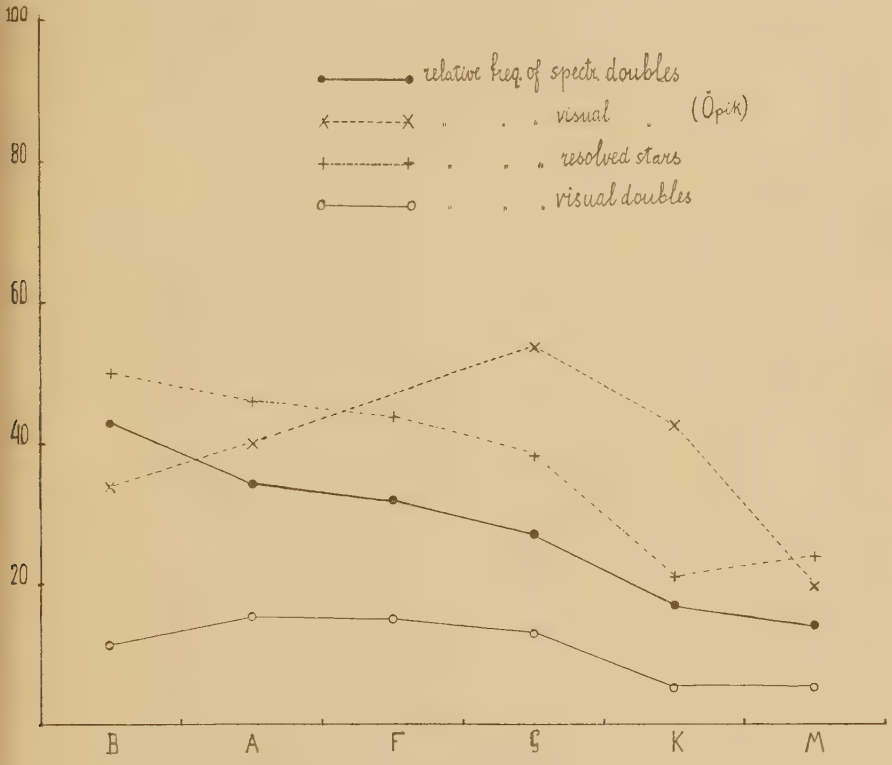


Fig. 1.—Relative frequency of visual doubles and spectroscopic ones and the frequency of resolved stars.

From the Tables V. and VI. it appears, however, that with those types also there is a regular increase of the percentage of spectroscopic doubles with the earlier types of the giant series.

Therefore we conclude that this effect is real. Considering the visual doubles we remark the increase of the percentage up to the A5–B8 stars. With the O stars, however, there is a decrease. This decrease, if real, may also be an effect of the high absolute luminosity of the O stars. Owing to their great distances a larger fraction of these double stars will have an angular separation, $< 0''{\cdot}50$, and will therefore be very incompletely observed. At present I have not tried to introduce

any correction for these larger distances. In the *B.A.N.*, 178, I gave a first solution for the distribution curve $\psi(L)$ of the distances $L = \log d$. This solution is probably entirely based on absolutely faint stars, and it is therefore doubtful if this curve can be applied to the stars of the giant series.

The distribution curves of Table V. have been graphically repre-

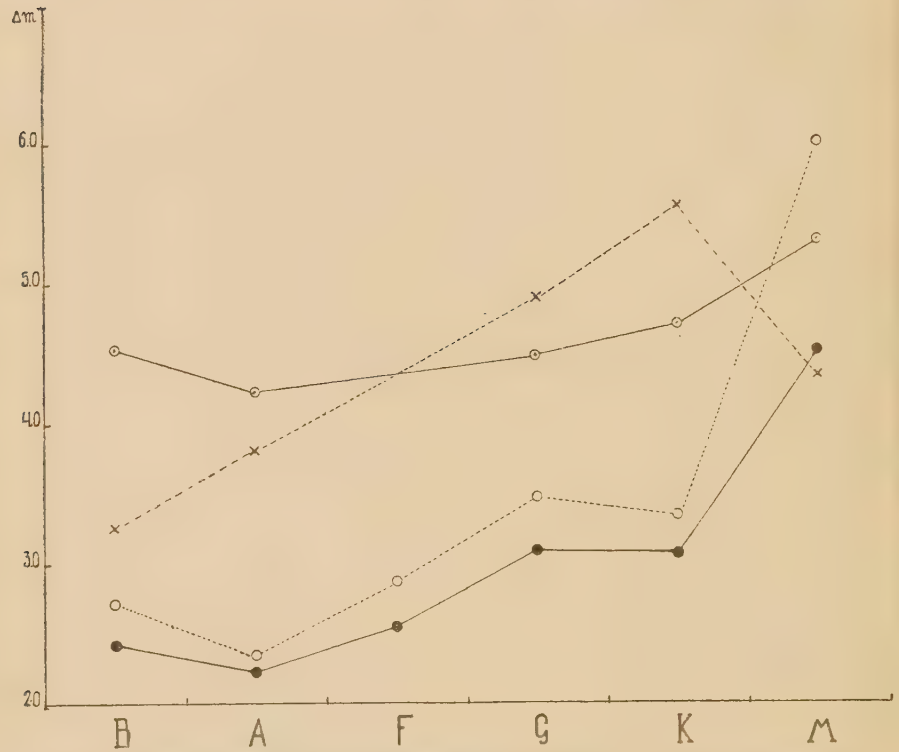


Fig. 2.—Mean values of Δm for different spectral types. Black dots represent the unweighted mean, open circles the weighted mean. Crosses and open circles with dots represent the values derived by Öpik for close and wide pairs.

sented in fig. 1. Instead of the numbers of stars which are to be considered as normal I give the percentages of the stars which in some way or other are double. In the same figure I also give the curve derived by Öpik.* His curve deviates widely from the ones derived by me (Öpik's values are expressed in the relative frequency of A double stars as unit).

Apparently the mean magnitude difference between the two components of a double star also varies with spectral type. In Table VIII. I have given this mean value $\overline{\Delta m}$ for the different spectral types.

* *Publ. Tartu*, tome xxv., No. 6, 1924.

Only those pairs are taken into account which fall within the limits of Table II. (*c*). Two series of values are given. The first values are simply the means of the observed values ; with the second set weights have been given to the different groups of stars. The intervals of d for the stars with large Δm are smaller than those with small Δm. The unweighted mean value of $\overline{\Delta m}$ is therefore always too small. The weights were chosen in such a way that all numbers were reduced to an equal interval of d. Accordingly the numbers with large Δm were multiplied by fractions, which are equal to the quotients of the total observed numbers up to the different limits of d. The details of the reductions appear in Table IX.

TABLE VIII.

Mean Values of Δm for Different Spectral Types.

Spectral Type.	O–B_7.	B_8–A_5.	F.	G.	K.	M.
Direct mean	2·42	2·21	2·55	3·10	3·10	4·53
Weighted mean	2·72	2·35	2·86	3·47	3·33	6·07
Öpik (close pairs)	3·26	3·80	..	4·90	5·57	4·35
Öpik (wide pairs)	4·54	4·22	..	4·49	4·70	5·35

TABLE IX.

Reduction to equal Intervals of d.

Δm. \ d.	<1″.	1″–2″.	2″–5″.	5″–10″.	10″–15″.	15″–30″.	Fraction.
0·0	45	16	12	10	2	7	
1·0	23	9	12	12	5		$\frac{92}{86}=1{\cdot}08$
2·0	6	7	12	8	8		1·08
3·0	6	10	16				$\frac{187}{142}\times 1{\cdot}08=1{\cdot}41$
4·0	2	8	14				1·41
5·0	2	4					$\frac{198}{132}\times 1{\cdot}41=2{\cdot}01$
6·0	1	6					2·01
7·0	0						..
8·0							

In the last line of Table VIII. I have entered the corresponding values of $\overline{\Delta m}$ as derived by Öpik.* The general run of his numbers is the same, but all his values are larger than mine. Considering fig. 2, where the different curves of Table VIII. are graphically represented, it is evident that with the earlier spectral types the mean magnitude difference

* *Loc. cit.*

decreases. The mean value of Δm which Öpik obtained for his close M-type pairs gives a large deviation, but the other three curves are very consistent. There is perhaps a faint indication that with the O–B8 stars the mean Δm increases again, but the evidence for this is certainly not convincing.

For the present I have not tried to find an explanation of the results here obtained, nor have they been compared with theory. It is, however, evident that if a star, after having once reached a position in the main sequence, proceeds, by emission of mass, along that sequence towards the later spectral types, then we must expect that the very high percentage of resolved stars which was found among the earlier types will also occur with the later types. This is what I would have anticipated when taking into account the results obtained in some previous papers.*

I have considered the possibility of extending the results obtained in the present paper to the fainter apparent magnitudes in order to get definite evidence as to the distribution of the double stars along the main sequence. With the spectroscopic doubles this will be almost impossible. In the first place, our knowledge concerning the radial velocities of the fainter stars is very small. In the second place, there would always be a danger, even if the fainter magnitudes were observed, of the numbers of spectroscopic doubles statistically being incomplete. According to formula (2),

$$K \propto (1 - e^2)^{\frac{1}{2}} \cdot P^{-1} \cdot M\left(\frac{\gamma}{1+\gamma}\right)^3,$$

i.e., the amplitude of variation of the radial velocity will be proportional to the mass of the star and to a function of the mass-ratio. With the later types of the main sequence the masses of the stars will most certainly decrease, and the same is the case with the mass-ratio. Circumstances are therefore very unfavourable for the detection of the dwarfs of later types which are spectroscopic doubles. So for the spectroscopic doubles an extension of our results to the absolutely faint stars is out of the question.

From a theoretical point of view I think that the high frequency of spectroscopic doubles with the O and B stars indicates that the same will be the case with the later type stars of the main sequence. There is, of course, also a possibility, that by a widening of the orbit, the spectroscopic doubles develop into visual ones.

With the visual doubles circumstances are more favourable. Here there actually is an opportunity for carrying on this investigation down to at least the 9th magnitude. In the Greenwich zone of the Astrographic Catalogue Dr. H. Groot † has collected all double stars down to $m = 12{\cdot}5$. A part of this zone is covered by the investigations of B. Lindblad ‡ and H. Petersson,§ who have determined the spectro-

* Comp. *B.A.N.*, **173**, 1929, and *M.N.R.A.S.*, May 1929.
† *M.N.R.A.S.*, **88**, 51–56, 1927 (Astrographic Cat., v., 150–189).
‡ *Nova Acta Sc. Upsala*, série iv., vol. vi., 1923.
§ *Meddelanden Upsala*, No. 29, 1927.

scopic parallaxes of a large number of stars, many being fainter than 9th magnitude. As the above mentioned publications contain the results obtained in complete areas of the sky, there can be no selection according to apparent magnitude or proper motion. By using this material I hope to be able to give shortly the results for a part of the main sequence also.

Concerning the mean magnitude difference between the components, one additional remark can be made which for the present, however, has an entirely hypothetical character. If in a colour-spectral type diagram we plot the absolute magnitudes (Table VII.) of the stars of different spectral types, and connect each point obtained in this way with a point of the main sequence having a magnitude difference equal to the mean magnitude difference $\overline{\Delta m}$ (see Table VIII.), then we obtain a set of lines converging almost to the same point of the main sequence.

This is the case both when my own values are used and also when the absolute magnitudes used by Öpik and his corresponding $\overline{\Delta m}$ for close or wide pairs are plotted.

With the B and A stars the points indicating the " mean companion " lie higher than the other ones. I do not give any details, as, owing to the large uncertainties of the determination of $\overline{\Delta m}$, there is much which is more or less hypothetical in the following. The distribution of the points indicating the " mean companion " along the main series may have some importance for cosmogonical problems. In the first place, the fact that the majority of the lines converge towards the same point may indicate that in the same interval of time wherein the main star passes along the whole giant series, the corresponding shift of the companion along the main sequence is but small. If from more complete material the fact stated above could be confirmed, we would perhaps obtain in this way a time scale by which the relative velocity of development along the giant and main series could be measured. In the second place, if it could be confirmed that the " mean companion " of the early type giants indicates an earlier state of the main sequence than that of the later giants, this would prove that with the earlier types double stars are still originating. The alternative would be that the high frequency of the double stars with A and B type giants is caused by the lower rate of the development of the double stars of these types as compared with that of the ordinary stars. As stated before, I do not give the above-mentioned points as definitely established facts. They may be, however, of cosmogenetical consequence, and may furnish interesting points for a future investigation in which stars with large Δm can more completely be taken into account.

Nova 2 Geminorum and a Dark Nebula. By T. E. Espin.

A letter was received early in 1928 January from Dr. Steavenson asking us to examine Nova 2 Geminorum, as he suspected nebulosity about it. The 24-inch Calver was used on the nights of January 26 and 27, and the following are the notes made on these nights.

January 26. A magnificent night, Vega very conspicuous, north, and ϵ, δ, η Canis Majoris equally so, south. The small stars given in the Greenwich diagram of the nova (*M.N.*, **72**, 516) were well seen. The nova was apparently the same as the other stars, but Mr. Milburn thought once or twice that it was slightly hazy, and estimated it 0·2 brighter than Steavenson's star marked X. I could, however, see no nebulosity about it.

January 27. It is again quite as clear as last night, though the definition is not so good. The nova was observed near the meridian, and all the stars on the Greenwich Chart were quite easy. It was not quite certain, but the nova was probably 0·1 fainter than X of Dr. Steavenson. The magnitude of X is provisionally 14·4 and Y 15·0, so that the nova would be 14·5 according to Dr. Steavenson's scale. No nebulosity was seen round the nova, and it and the *comes* seemed equally sharp. Using the sweeping power made by Calver, where the field lens is 2 inches in diameter and the field about 45′, and sweeping rapidly backwards and forwards, it was obvious that the field of the nova is occupied by a dark nebula, very like the one found by me in Perseus (*M.N.*, **58**, 334, 1898). It lies within the B.D. map of zone $+32°$, $6^h\ 44^m$ to $6^h\ 48^m$. It is oval, the longer axis being marked by B.D. $+32°{\cdot}1437$ $6^h\ 45^m\ 27^s{\cdot}9 + 32°\ 40'{\cdot}3$ (1855) at one end, and B.D. $+31°{\cdot}1440$ $6^h\ 45^m\ 53^s{\cdot}7 + 31°\ 57'{\cdot}5$ (1855) at the other, and the nova is fairly central. The diameters of the object are 40′ by 20′ approximately. The darkest portion is north of the nova. It seemed a dark reddish colour.

Some further light is thrown upon this object by the *Carte Photographique du Ciel*, No. 46, zone $+33°$, made at Uccle by J. Delvosal 1910 March 2, or some two years before the outburst. The nova is shown as a star of magnitude 14·2, star X also, and assumed as 14·4, while star Y is not visible, falling on the line -45 of the Chart, but would in all probability be too faint to be seen in the reproduction. The absorption north of the nova is well seen in the zones marked -30 to -45 in declination, and R.A. -10 to $+5$, thus agreeing with the note made at the telescope that "the darkest portion is N. of the nova."

Observations of (*Ch.* 678) *U Persei*, 1924–29. By A. N. Brown, M.A.

The 249 observations of U Persei (the Harvard designation of which is 015254) recorded below are in continuation of those published in

M.N., **84**, 615–620 (1924). They have been plotted, and light curves drawn, from which the following tables have been compiled :—

Maxima.				Minima.			
No.	Date.	Julian Date.	Mag.	No.	Date.	Julian Date.	Mag.
18	1924 Nov. 9	4099	8·3	18	1925 Mar. 2	4212	9·6
19	1925 July 2	4334	8·3	19	1926 Apr. 3	4609	9·8
20	1926 June 20	4687	8·2	20	1927 Jan. 17	4898	10·0
21	1927 Apr. 27	4998	8·4	21	1927 Nov. 23	5208	10·1
22	1928 Apr. 4	5341	7·7	22	1928 Oct. 14	5534	10·4
23	1929 Mar. 23	5694	7·8	23	1929 Aug. 23	5847	10·3

The above phases are numbered in continuation of those previously given. In the table which follows there are shown the intervals in days between (*a*) successive maxima, (*b*) successive minima ; and the duration, also in days, of (*c*) the rise from minimum to maximum, and (*d*) the fall from maximum to minimum :—

No.	(*a*).	(*b*).	(*c*).	(*d*).	No.	(*a*).	(*b*).	(*c*).	(*d*).
18	340	303	190	113	21	311	310	100	210
19	235	397	122	275	22	343	326	133	193
20	353	289	78	211	23	353	313	160	153

Although the intervals between successive phases, as given here and previously, differ very considerably from each other, the mean period shows little change. That obtained from the maxima and minima under consideration is 322·75 days as compared with 320·6 days, the period derived from all my observations since 1909.

As shown above, the light has varied during these years between $7^{m}\cdot7-8^{m}\cdot4$ and $9^{m}\cdot6-10^{m}\cdot4$ (Hagen's scale), and it is noticeable that after a succession of bright minima extending from 1922 to 1926, since the latter year the light at minimum has again been estimated as $10^{m}\cdot0$, or fainter, as it was in every year before 1922.

In the third column of the observations here recorded the comparisons are given exactly as they were made. Thus the first entry is intended to show that the light of the Variable was considered to be midway between that of each of the comparison stars 7 and 13, while the second indicates that the Variable was estimated as being $0^{m}\cdot2$ fainter than 7, and $0^{m}\cdot4$ brighter than 13. In cases where the two methods of comparison are used upon the same occasion, equal weight is given to each result. The comparison stars are numbered as in the catalogue of Hagen's *Atlas Stell. Var.*, series iii., and to preserve continuity Hagen's own magnitudes have again been adopted, though in the table which follows the observations the Harvard values also are given. These have been obtained from *Specola Astro. Vaticana*, xi., " Aggiunte al Catalogo dell' A.S.V.," p. 247.

The same telescopes have been used as previously, viz. 5-inch and 3-inch refractors with powers varying from 50 to 120.

The Observations.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1924			
June 13	3950	7(1)v(1)13	8·9
20	3957	7−2, 13+4	8·8
July 10	3977	7+1	8·5
13	3980	3−4, 7+2	8·4
19	3986	3−5, 7+1	8·5
24	3991	7(1)v(4)13	8·7
29	3996	7+1	8·5
Aug. 2	4000	7+1	8·5
7	4005	3−5, 7+1	8·5
12	4010	3(5)v(1)7	8·5
19	4017	3−5, 7+1	8·5
25	4023	3−4, 7+2	8·4
31	4029	3−5, 7+1·5	8·5
Sept. 8	4037	3−5, 7+1	8·5
22	4051	3−4, 7+2	8·4
27	4056	3−4, 7+2	8·4
Oct. 14	4073	3−4, 7+2	8·4
23	4082	3−4, 7+2·5	8·4
Nov. 3	4093	3(1)v(1)7	8·3
16	4106	3−3, 7+3	8·3
27	4117	3−4, 7+2	8·4
Dec. 5	4125	7+1·5	8·5
14	4134	3−4, 7+2	8·4
24	4144	3−5, 7+1	8·5
1925			
Jan. 4	4155	7−1	8·7
6	4157	7−1	8·7
23	4174	7(1)v(2)13	8·8
Feb. 1	4183	7−2, 13+4	8·8
6	4188	7−4, 13+2	9·0
15	4197	13+1	9·1
18	4200	13+1	9·1
22	4204	13(2)v(1)17	9·4
Mar. 2	4212	16−1, =17, 21+2, 24+1	9·5
8	4218	17−1, 21+1, 24+1	9·6
12	4222	13−3, 17+1, 24+2	9·5

Date.	Julian Date.	Comparisons.	Ded. Mag.
1925			
Mar. 18	4228	13(3)v(1)17, 16(1)v(1)17	9·4
21	4231	13(4)v(1)17, 17+0·5	9·4
28	4238	13−2, 17+1	9·4
Apr. 3	4244	13−2, =16, 17+1	9·4
8	4249	13−2, 16−1, 17+1	9·4
12	4253	13(2)v(1)17	9·4
21	4262	13−2, 16+1, 17+2	9·3
May 12	4283	13(2)v(1)17	9·4
20	4291	=13, 17+3	9·2
25	4296	7−5, 13+1	9·1
June 1	4303	7(2)v(1)13	9·0
8	4310	7−4, 13+2	9·0
17	4319	7(1)v(2)13	8·8
29	4331	3(1)v(1)7	8·3
July 7	4339	3−3, 7+3	8·3
11	4343	3−4, 7+2	8·4
14	4346	3−4, 7+2	8·4
18	4350	3−5, 7+1	8·5
Aug. 2	4365	3−4, 7+2	8·4
7	4370	3−5, 7+1	8·5
17	4380	3−5, 7+1	8·5
29	4392	3−5, 7+1	8·5
Sept. 4	4398	3−4, 7+2	8·4
11	4405	=7	8·6
14	4408	3−5, 7+1	8·5
18	4412	7+1	8·5
23	4417	=7	8·6
26	4420	7−1, 13+5	8·7
Oct. 3	4427	7−1, 13+5	8·7
8	4432	7−2, 13+4	8·8
14	4438	7−2, 13+4	8·8
23	4447	7−4, 13+2	9·0
Nov. 4	4459	7−5, 13+1	9·1
9	4464	7−3, 13+3	8·9
12	4467	7−2, 13+4	8·8
15	4470	7−2, 13+4	8·8

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1925			
Nov. 19	4474	7−2, 13+4	8·8
25	4480	7(1)v(1)13	8·9
Dec. 2	4487	7−3, 13+3	8·9
5	4490	7−4, 13+2	9·0
9	4494	7−4, 13+2	9·0
17	4502	7−4, 13+2	9·0
31	4516	7−5, 13+2	9·1
1926			
Jan. 4	4520	7−5, 13+1	9·1
11	4527	13(2)v(1)16	9·3
19	4535	=16, 17+1	9·4
Feb. 13	4560	13(2)v(1)17, =16	9·4
24	4571	16+1, 17+2	9·3
Mar. 9	4584	16+1, 17+2	9·3
13	4588	13(2)v(1)17, 16+1	9·4
17	4592	13(2)v(1)17	9·4
21	4596	13−3, 16−2, =17	9·5
26	4601	17−2, =24	9·7
30	4605	17−2, =23, 24−1	9·7
Apr. 5	4611	17−3, =23, 24−1	9·8
11	4617	17−2, 23+0·5, =24	9·7
15	4621	=17, 23+2, 24+2	9·5
17	4623	13−1, =16, 17+1	9·4
May 3	4639	7−5, 13+1	9·1
13	4649	7−4, 13+2	9·0
June 5	4672	3−4, 7+2	8·4
22	4689	3−2, 7+4	8·2
26	4693	3−2, 7+4	8·2
July 13	4710	3−4, 7+2	8·4
15	4712	3−4, 7+2	8·4
30	4727	3−4, 7+2	8·4
Aug. 4	4732	3−5, 7+1	8·5
7	4735	3−5, 7+1	8·5
14	4742	7(1)v(5)13	8·7
21	4749	7(1)v(5)13	8·7
26	4754	7−2, 13+4	8·8
Sept. 4	4763	7−3, 13+3	8·9
10	4769	7(4)v(3)13	8·9
1926			
Sept. 13	4772	7−4, 13+2	9·0
19	4778	7(4)v(3)13	8·9
26	4785	7(3)v(4)13	8·9
30	4789	7−4, 13+2	9·0
Oct. 8	4797	7−4, 13+2	9·0
18	4807	7−4, 13+2	9·0
26	4815	7−5, 13+1	9·1
30	4819	7−5, 13+1	9·1
Nov. 7	4827	7−5, 13+1	9·1
Dec. 2	4852	13−3, 16−1, =17	9·5
8	4858	=17	9·5
20	4870	17−1	9·6
29	4879	23(1)v(2)29, 24−1	9·8
1927			
Jan. 4	4885	23+1, 24+1	9·6
21	4902	=29	10·0
Feb. 2	4914	17−2, =21, =24, 29+2	9·7
4	4916	17(2)v(1)23	9·6
27	4939	16−1, =17	9·5
Mar. 7	4947	13−1, 16+1, 17+2	9·3
23	4963	7(3)v(2)13	9·0
27	4967	7(2)v(1)13	9·0
Apr. 1	4972	7−2, 13+4	8·8
7	4978	3(5)v(1)7	8·5
15	4986	3−4, 7+2	8·4
19	4990	3−4, 7+2	8·4
26	4997	3−4, 7+2	8·4
30	5001	3−4, 7+2	8·4
May 3	5004	3(2)v(1)7	8·4
8	5009	3−4, 7+2	8·4
11	5012	3−3, 7+3	8·3
June 3	5035	3−5, 7+1	8·5
21	5053	3−5, 7+1	8·5
July 5	5067	3(5)v(1)7	8·5
26	5088	7+1	8·5
Aug. 2	5095	3−5, 7+1	8·5

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1927			
Aug. 11	5104	7+1	8·5
16	5109	=7	8·6
23	5116	7(1)v(3)13	8·8
Sept. 2	5126	7(2)v(1)13	9·0
7	5131	7(2)v(1)13	9·0
22	5146	13+1	9·1
25	5149	7−5, 13+1	9·1
29	5153	13+1	9·1
Oct. 3	5157	13(2)v(1)17, =16	9·4
17	5171	=24	9·7
18	5172	21−1, 22−1, 23−1, 24−1	9·8
23	5177	23(2)v(1)33	10·0
31	5185	23(1)v(1)28, 24−1	9·8
Nov. 11	5196	23−3, =29, 33+1	10·0
16	5201	23−2, 24−2, 33+1, 34+3	10·0
24	5209	29(1)v(2)34	10·1
Dec. 20	5235	23(3)v(1)28, 24(3)v(1)29	9·9
1928			
Jan. 9	5255	17(1)v(1)23	9·6
16	5262	17−0·5	9·6
22	5268	13(2)v(1)17	9·4
27	5273	13+1	9·1
Feb. 1	5278	7(3)v(1)13	9·1
13	5290	7−3, 13+3	8·9
19	5296	7+1	8·5
22	5299	3−4, 7+2	8·4
25	5302	3−3, 7+3	8·3
Mar. 4	5310	3−2, 7+4	8·2
10	5316	3−2, 7+4	8·2
30	5336	3+2	7·8
Apr. 4	5341	3+3	7·7
18	5355	3+2	7·8
28	5365	3−3, 7+3	8·3
May 5	5372	3−4, 7+2	8·4
14	5381	3−5, 7+1	8·5

Date.	Julian Date.	Comparisons.	Ded. Mag.
1928			
May 18	5385	7+05	8·6
25	5392	=7	8·6
June 10	5408	7−1, 13+5	8·7
16	5414	7−5, 13+1	9·1
22	5420	7−2, 13+4	8·8
July 14	5442	7(2)v(1)13	9·0
17	5445	7(2)v(1)13	9·0
21	5449	7(3)v(1)13	9·1
28	5456	7−5, 13+1	9·1
Aug. 8	5467	7−4, 13+1	9·1
18	5477	=13	9·2
28	5487	13(2)v(1)17, =16	9·4
Sept. 4	5494	16(2)v(1)21, 22+1, 17(1)v(1)23, =24	9·6
9	5499	16(3)v(1)21, 22+1, 17(2)v(1)23, 24+1	9·6
17	5507	23(1)v(2)29, 24(1)v(1)33	9·9
20	5510	23−1, 24(1)v(2)28, 29+2	9·8
Oct. 1	5521	33(1)v(1)34, 37+3, 39+3	10·2
4	5524	33−1, 34+2, 37+2, 39+2	10·2
11	5531	33(2)v(1)37, 34(2)v(1)39	10·4
17	5537	33(2)v(1)37, 34(1)v(1)37, 39+1	10·4
21	5541	33(1)v(2)37, 34+3	10·1
25	5545	23(1)v(1)28, 24−1	9·8
Nov. 7	5558	16−1, 17(1)v(1)23, =24	9·6
9	5560	16(2)v(1)23, 17−1, 24+1	9·6
17	5568	16−1, =17	9·5
25	5576	=13	9·2
Dec. 5	5586	7(4)v(1)13	9·1
8	5589	13+1	9·1

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1928			
Dec. 17	5598	7(1)v(1)13	8·9
29	5610	7−3, 13+3	8·9
1929			
Jan. 12	5624	7(1)v(5)13	8·7
Feb. 3	5646	3(1)v(5)7	8·1
12	5655	3+1	7·9
28	5671	3+1	7·9
Mar. 2	5673	3+1	7·9
6	5677	3+1	7·9
10	5681	3+1·5	7·9
20	5691	3+2	7·8
29	5700	3+2	7·8
Apr. 1	5703	3+2	7·8
5	5707	3+1	7·9
18	5720	3(1)v(2)7	8·2
May 2	5734	3(4)v(1)7	8·5
9	5741	3(5)v(1)7	8·5
12	5744	7+1	8·5
15	5747	=7	8·6
19	5751	=7	8·6
June 10	5773	7(3)v(2)13	9·0
18	5781	7(5)v(1)13	9·1
24	5787	=13	9·2
July 8	5801	16−2, 17(1)v(1)21	9·6
11	5804	16(3)v(1)23, 22+1	9·6
1929			
July 14	5807	17(2)v(1)23, 21+1, 22+1, 24+1	9·6
17	5810	17(2)v(1)23, 22+1, 24+1	9·6
24	5817	17−2, =23, =24, 21+1, 22+1	9·7
29	5822	23(2)v(1)29	9·9
Aug. 7	5831	23(3)v(1)34	10·2
13	5837	23(3)v(1)37, =28, 33+1, 34+1	10·1
25	5849	34(1)v(2)37, 39+3	10·3
Sept. 1	5856	23(2)v(1)34	10·1
3	5858	23(2)v(1)29, 24−3	10·0
6	5861	23(1)v(1)29, 24−2	9·9
11	5866	23(1)v(1)29	9·9
23	5878	16−1, 17+1	9·5
26	5881	13(1)v(1)17	9·4
Oct. 9	5894	7(5)v(1)13	9·1
16	5901	7(3)v(1)13	9·1
26	5911	7(2)v(1)13	9·0
Nov. 3	5919	7(1)v(3)13	8·8
9	5925	7−1, 13+5	8·7
12	5928	3−5, 7+1	8·5
20	5936	7+1	8·5

The Comparison Stars Used.

Hagen's Number.	Hagen's Magn.	Harvard Magn.	Hagen's Number.	Hagen's Magn.	Harvard Magn.
3	8·0	7·3	24	9·7	10·0
7	8·6	8·6	28	9·9	10·2
13	9·2	9·5	29	10·0	10·2
16	9·4	9·7	33	10·1	10·4
17	9·5	9·8	34	10·3	10·5
21	9·7	10·0	37	10·5	10·8
22	9·7	10·0	39	10·5	10·8
23	9·7	10·0			

Brackenhurst,
Bucklebury Common, Berks :
1929 *November.*

Observations of Comet Neujmin (1929*b*) *made at Helwan Observatory.*
By M. R. Madwar, B.Sc., Ph.D.

The following positions have been derived from plates taken with the Reynolds 30-inch reflector.

Comet Neujmin (1929*b*).

Date.	G.M.T.	Exposure.	R.A. 1929·0.	log $p\Delta$.	Decl. 1929·0.	log $p\Delta$.	Corr. for Earth's Aberr. R.A.	Corr. for Earth's Aberr. Dec.
	h m s	m	h m s		° ′ ″		s	″
1929 Oct. 2	20 9 18	60	21 16 18·15±0·01	9·362	−15 47 47·3±0·27	0·783	+0·83	+4·0
3	19 32 37	60	21 16 57·59±0·02	9·196	−15 46 56·8±0·59	0·791	+0·82	+3·7
5	19 38 24	60	21 18 22·26±0·03	9·263	−15 44 48·6±0·50	0·788	+0·79	+3·6

The positions above are referred to the equator and equinox of 1929·0.
The true mean places for 1929·0 may be obtained by

(1) Applying the correction for parallax,
(2) Applying the correction for the earth's aberration,
(3) Ante-dating by the light time.

The Orbit of Comet Pons-Coggia-Winnecke-Forbes.
By A. C. D. Crommelin, D.Sc.

Mons. F. Quénisset gave a description on page 166 of the present volume of *M.N.* of the images that he obtained of the above comet on plates taken at Suhalia, Switzerland, on 1928 October 25, for the purpose of photographing the zodiacal light. M. Quénisset sent the original negatives and also glass positives of his plates to the Royal Observatory, Greenwich, where Mr. C. R. Davidson has measured them micrometrically. The star images in the first plate are much better than in the other, so this has alone been used. In spite of the small scale of the photographs (focal length 5·3 inches) Mr. Davidson deduced the probable error of the resulting position of the comet as under 10″ in each coordinate; since the positions are a month earlier than any other observations of the comet (Mr. Yamasaki's estimated position on October 26.81, though obviously belonging to the comet, is not exact enough to use) they afford valuable material for improving the elements of the comet at the apparition of 1928.

The second and third positions that I have used in deducing a new orbit were photographic positions obtained at Algiers and Johannesburg respectively. I have used the photograph of December 16 in preference to that of December 20, as the comet was becoming rather difficult on the latter date. The following are the positions used, corrected for parallax :—

U.T. 1928.	Light-time.	R.A. 1928·0.	Decl. 1928·0.	Parx. Corr. applied.
Oct. 25·15417	·00315	164° 13′ 48″·15	+11° 32′ 27″·87	−10″·35+10″·5
Nov. 24·21465	·00540	184 15 33·58	−23 58 45·57	− 5·42+ 7·43
Dec. 16·05899	·00691	198 6 2·52	−35 0 54·06	− 7·23− 0·76

The orbit deduced by Mr. H. E. Wood, which is given in *M.N.*, **89**, 556 (1929), was taken as a basis for differential correction. The correction in the period is so large that two successive applications of the formulæ were required. The following elements were obtained :—

T 1928 Nov. 4·95184 U.T.

ω	195° 52′ 30″·34	1928·0
Ω	250 3 59·20	1928·0
i	28 53 50·28	1928·0
ϕ	66 46 59·67	
log a	0·9637422	
log q	9·8721195	
Period	27·90058 years	

Gaussian equations :—

$$x = [9{\cdot}9498066]\, r \sin (v + 173^\circ\, 22'\, 26''{\cdot}58)$$
$$y = [9{\cdot}9913840]\, r \sin (v + 77\ 29\ 7{\cdot}62)$$
$$z = [9{\cdot}6948313]\, r \sin (v + 146\ 49\ 12{\cdot}89)$$

The following are the tabular places computed from these elements for the times of the observations :—

164° 13′ 49″·25	+ 11° 32′ 27″·96
184 15 31·84	− 23 58 45·89
198 5 59·52	− 35 0 52·57

It will be noticed that the period found is in good agreement with the value obtained from the perturbations from 1873 to 1928, which was 27·8969 years (*M.N.*, **89**, 560, 1929).

Note on the Rotation of the Galactic System. By Sir Frank Dyson.

In Nos. 120 and 132 of the *Bulletin of the Astronomical Institutes of the Netherlands* Mr. Oort has published two important papers on the rotation of the galactic system. For the proper motion, due to rotation round a centre in longitude l_0, he finds the formula

$$\mu_l = \frac{A}{4{\cdot}74} \cos 2(l - l_0) \cos b + \frac{B}{4{\cdot}74} \cos b,$$

where

$$A = \frac{V}{4R}\left(1 - \frac{R}{K}\frac{\partial K}{\partial R}\right) \text{ in km./sec. per parsec.}$$

and

$$B = A - \frac{V}{R},$$

and R = distance from Sun to centre of Galactic System,
K = total gravitational force at the Sun's distance,
V = circular velocity in km./sec.

Mr. Oort has determined the values of $\frac{A}{4{\cdot}74}$ and $\frac{B}{4{\cdot}74}$ from the proper motions of groups of distant stars and from the stars of Boss's Catalogue.

In this note, the stars in Boss's Catalogue of types Bo B5, B8, B9, Ao, and A2–A5, which lie within 30° of the galactic plane, have been considered. The only stars omitted are nine stars of types B8 and Ao in the Draper Catalogue brighter than 2·0. The Pleiades and the Taurus clusters have been treated as one star each. The stars are grouped for each 10° of longitude, and the mean proper motions for each group are given in the following table :—

TABLE I.

Mean Proper Motions in Galactic Longitude and Latitude of Stars Bo–B5, B8–Ao, A2–A5. Unit ″·0001.

	Bo–B5.			B8–Ao.			A2–A5.		
l.	μ_l.	μ_b.	No.	μ_l.	μ_b.	No.	μ_l.	μ_b.	No.
° °									
0–9	−120	− 88	4	−105	−128	13	+ 29	−119	8
10–19	− 58	− 10	4	−155	− 1	15	+ 65	−117	11
20–29	−121	− 80	9	− 54	− 59	18	+257	−142	15
30–39	− 61	− 17	13	− 35	− 40	15	+140	− 38	12
40–49	− 39	− 67	14	+ 80	− 99	24	+ 66	+ 19	8
50–59	+ 43	− 68	13	+ 44	− 97	26	+508	+ 69	9
60–69	0	− 16	15	+118	−135	22	+459	− 82	10
70–79	+ 68	− 28	11	+217	− 78	20	+146	−236	10
80–89	+119	− 58	13	+225	− 48	26	+ 53	−229	21
90–99	+206	− 40	7	+284	− 45	29	+523	−100	17
100–109	+ 35	+ 23	6	+263	− 70	24	+145	− 60	14
110–119	+266	− 21	16	+398	−119	15	+274	−173	9
120–129	+229	− 29	14	+274	− 67	23	+209	− 65	22
130–139	+415	−107	5	+340	− 95	21	+306	−183	10
140–149	+144	− 14	8	+315	− 70	23	+287	−135	6
150–159	+171	− 18	14	+247	− 98	24	+390	−134	13
160–169	+124	− 25	21	+214	−106	20	+218	−161	12
170–179	+ 26	− 20	25	+ 83	−128	17	+209	−179	10
180–189	+ 9	− 68	16	+ 6	− 84	20	+115	− 22	8
190–199	− 13	− 79	12	+ 90	−179	14	+ 45	−219	8
200–209	− 88	− 43	31	− 16	− 59	17	− 36	−128	8
210–219	− 48	−144	25	− 85	−182	24	− 25	−210	13
220–229	− 55	−129	23	− 69	−178	22	−487	−135	11

TABLE I.—*continued.*

	B0–B5.			B8–A0.			A2–A5.		
l.	μ_l.	μ_b.	No.	μ_l.	μ_b.	No.	μ_l.	μ_b.	No.
° °									
230–239	− 72	−178	32	− 86	−171	29	−303	−209	7
240–249	−138	−190	20	−283	−174	19	−546	−556	10
250–259	−123	−140	11	−304	−136	25	−431	−239	11
260–269	−309	−222	25	−336	−236	21	−538	−204	12
270–279	−296	−209	17	−290	−279	22	−130	−265	11
280–289	−354	−143	22	−362	− 85	11	−784	−400	7
290–299	−328	−180	13	−391	− 74	28	−769	−229	10
300–309	−284	− 59	22	−360	−164	13	−297	−265	7
310–319	−306	−105	26	−378	−137	18	−279	−183	7
320–329	−246	− 87	15	−340	−211	21	−314	−216	16
330–339	−150	+ 32	12	−166	−160	17	−132	−101	13
340–349	−137	+ 17	7	−165	− 74	22	−281	− 86	14
350–359	− 92	− 85	4	−219	−153	18	−187	+ 12	6

Analysis of this table gives—

$$\text{B0–B5}\quad \mu_l = -0''.0044 - 0''.0233\cos(l + 59°.3) + 0''.0026\cos 2(l - 336°) + 0''.0058\cos 3(l + 1°)$$

$$\text{B8–A0}\quad \mu_l = -0''.0028 - 0''.0324\cos(l + 62°.2) + 0''.0019\cos 2(l - 327°) + 0''.0034\cos 3(l + 8°)$$

$$\text{A2–A5}\quad \mu_l = -0''.0030 - 0''.0418\cos(l + 79°.0) + 0''.0104\cos 2(l - 8°) - 0''.0057\cos 3(l + 15°)$$

Examination of Table I. shows clearly that the irregularities in the mean proper motions of the A2 A5 stars are too great for any useful results to be derived from them. The longitude of the apex derived from them is also considerably away from the accepted value.

The solar motion terms, which are in good accordance, were taken out of the B0–B5 and B8–A0 groups, and the remainders combined according to the number of stars in each 10° of longitude (Table II., column 8). For the combined group, analysis gives

$$-0''.0036 + 0''.0020\cos 2(l - 330°) + 0''.0046\cos 3(l + 5°).$$

As cos b has not been taken into account, the first two coefficients may be written

$$-0''.0038\cos b + 0''.0021\cos 2(l - 330°)\cos b.$$

This is in good accordance with Oort's results and the phase of the term in $2l$ is quite satisfactory. but the term in $3l$ is somewhat disconcerting. It arises, I think, from the peculiar motions of the stars, rather than from systematic errors in the observations, but indicates that the term in $2l$ is liable to some uncertainty. On the other hand, the agreement of the phase with the supposed centre of the galactic system is better than might have been anticipated.

Fig. 1 shows the observations. The two curves drawn are for

$$+ ''\cdot0020 \cos 2(l - 330°),$$

and for the third harmonic

$$+ ''\cdot0046 \cos 3(l + 5°).$$

It seems likely that a good result would be obtained by using fainter stars of the same types of spectrum, within 30° of the galactic plane over the two hemispheres. The smaller proper motions and greater number of these stars should diminish the irregularities in the mean values of μ_l for the several groups. The value of $\frac{A}{4\cdot7\dagger}$ obtained from the radial velocities is ″·0040.

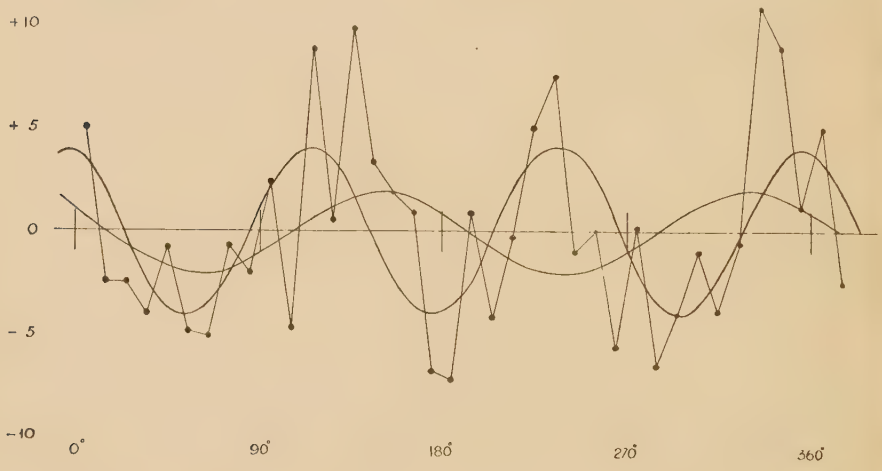


Fig. 1.

Galactic Latitude.—If X, Y, Z be the rectangular components of the solar motion referred to the galactic plane, the proper motions in longitude give

$$X \sin l - Y \cos l = \mu_l$$

from which X, Y are obtained, and the proper motions in latitude give

$$(X \cos l + Y \sin l) \sin b - Z \cos b = \mu_b.$$

In Table II. the value of b is given for each group. The quantities $(X \cos l + Y \sin l) \sin b$ are then computed and subtracted from the corresponding values of μ_b. From the means for the 36 groups the values of $-Z \cos b$ are calculated for the B0–B5, B8 A0, and A2 A5 stars. They are found to be $-''\cdot0084$, $-''\cdot0121$, and $-''\cdot0170$ respectively for the three groups of stars. These are taken out and the differences are given in the second column of Table II. for each group of stars.

TABLE II.

Mean Galactic Latitudes of Groups and Residuals after Solar Motion has been taken out. Unit ″·0001.

	Bo–B5.		B8–Ao.		A2–A5.		Bo–Ao.	
l.	*b.*	μ_b.	*b.*	μ_b.	*b.*	μ_b.	μ_l.	μ_b.
° °	°		°		°			
0–9	−11·0	+ 36	−7·6	+ 32	+ 9·8	− 20	+ 14	+ 33
10–19	+13·2	+ 23	+5·6	+ 91	+ 9·6	− 17	− 60	+ 77
20–29	− 0·5	+ 6	−3·9	+ 83	− 0·9	+ 34	− 60	+ 57
30–39	+ 8·4	+ 32	+4·0	+ 58	+10·1	+ 65	− 75	+ 46
40–49	+ 1·3	+ 12	+1·5	+ 14	+ 5·0	+159	− 45	+ 13
50–59	+ 2·4	+ 7	+0·8	+ 20	+ 9·4	+190	− 85	+ 16
60–69	− 6·2	+ 99	−0·8	− 13	+10·0	+ 42	− 87	+ 33
70–79	+ 3·9	+ 44	−1·3	+ 48	0·0	− 66	− 44	+ 46
80–89	− 7·5	+ 43	+7·0	+ 51	+ 2·3	− 64	− 56	+ 48
90–99	− 2·7	+ 50	−0·8	+ 78	+ 7·6	+ 64	− 12	+ 72
100–109	− 3·6	+111	+0·6	+ 50	− 5·2	+108	− 83	+ 62
110–119	− 3·1	+ 64	−0·6	+ 2	− 2·5	− 7	+ 53	+ 34
120–129	− 9·0	+ 52	−0·4	+ 54	+11·0	+137	− 31	+ 53
130–139	− 9·6	− 32	−3·7	+ 20	− 0·7	− 16	+ 62	+ 7
140–149	−10·3	+ 53	−6·6	+ 34	− 0·3	+ 34	− 2	+ 39
150–159	− 5·5	+ 54	−6·1	+ 3	− 7·6	− 8	− 17	+ 22
160–169	−12·4	+ 24	+2·2	+ 24	− 0·6	+ 5	− 27	+ 24
170–179	−16·3	+ 11	−5·3	− 32	+ 3·7	+ 17	−104	− 6
180–189	−12·2	− 28	+6·2	+ 69	+ 1·6	+160	−110	+ 26
190–199	− 2·6	− 5	0·0	− 78	−11·8	−134	− 27	− 44
200–209	−10·1	0	−5·7	+ 30	+ 8·8	+104	− 77	+ 11
210–219	− 8·4	− 94	−7·3	−102	−11·5	−116	− 39	− 98
220–229	− 2·7	− 56	−6·4	− 92	− 5·6	+ 1	+ 14	− 73
230–239	− 3·7	−108	−6·6	− 87	+ 1·1	− 33	+ 38	−100
240–249	− 3·7	−119	+1·2	− 48	+ 4·4	−367	− 46	− 84
250–259	− 0·1	− 56	−1·4	− 21	+ 5·4	− 52	− 36	− 32
260–269	− 0·1	−138	+1·4	−111	− 0·6	− 35	− 92	−126
270–279	+ 4·7	−117	+2·9	−151	− 3·5	− 98	− 34	−135
280–289	+11·5	− 46	−2·2	+ 33	+ 2·0	−231	−101	− 20
290–299	+ 5·7	− 94	+6·9	+ 50	− 7·3	− 47	− 76	+ 4
300–309	+ 2·1	+ 24	+5·3	− 46	+ 8·3	−139	− 46	− 2
310–319	+ 6·1	− 27	−0·4	− 15	− 5·0	+ 7	− 75	− 22
320–329	+ 8·1	− 18	−5·4	− 76	+ 2·3	− 58	− 42	− 52
330–339	− 4·4	+127	−6·1	− 19	+ 1·9	+ 58	+ 72	+ 41
340–349	− 1·7	+104	+1·4	+ 41	− 6·7	+128	+ 52	+ 56
350–359	+ 6·5	− 33	−3·4	− 16	+ 0·4	+179	− 25	− 19

Each of these groups shows very clearly a single period in l.

$$\begin{aligned} &\text{Bo–B5 gives } + ''\cdot0018 \cos l + ''\cdot0067 \sin l, \\ &\text{B8–Ao} \quad ,, \quad + ''\cdot0018 \cos l + ''\cdot0054 \sin l, \\ &\text{A2–A5} \quad ,, \quad + ''\cdot0024 \cos l + ''\cdot0079 \sin l. \end{aligned}$$

The group A2–A5 is very irregular and has not been considered further. But the two groups Bo–B5 and B8–Ao have been combined for each 10° according to the number of observations shown in Table I. and give the last column of Table II. From this is derived the result Bo–Ao $+ ''\cdot0020 \cos l + ''\cdot0060 \sin l$.

FIG. 2.

Following Oort's treatment but with slightly different co-ordinates for the Galactic Pole, viz. R.A. $12^h\ 41^m\ 20^s$, Dec. 27°·3, we find

$$\begin{aligned} \epsilon_1 &= 61°\cdot0 \qquad l_1 = 153°\cdot5 \\ \epsilon_2 &= 62°\cdot5 \qquad l_2 = 180° \end{aligned}$$

for the inclinations and longitudes of the nodes of the ecliptic and equator on the galactic plane. The above quantity may be equated to

$$\Delta p \sin 61° \cos (l - 153°\cdot5) - (\Delta\epsilon + \Delta\lambda) \sin 62°\cdot5 \cos (l - 180°)$$

where Δp is a correction to the luni-solar precession,

$\Delta\epsilon$,, ,, ,, planetary precession,

and $\Delta\lambda$ is a correction required to the motion of the equinox adopted by Newcomb. In order to show the smallness of the coefficient of Δp the numerical results are given in detail.

$$\cdot390\Delta p \sin l = + ''\cdot0060 \sin l$$

and

$$\{\cdot887(\Delta\epsilon + \Delta\lambda) - \cdot783\Delta p\} \cos l = + ''\cdot0020 \cos l,$$

or $$\Delta p = +''\cdot 015$$
$$\Delta\epsilon + \Delta\lambda = +''\cdot 014.$$

Oort from a more extensive discussion finds

$$\Delta p = +''\cdot 0113 \quad \text{and} \quad \Delta\epsilon + \Delta\lambda = +''\cdot 0117.$$

A rough reduction to the mean of the corrections of Raymond and of Boss and Jenkins reduces the quantity $+''\cdot 0020 \cos l + ''\cdot 0060 \sin l$ to $+''\cdot 0019 \cos l + ''\cdot 0055 \sin l$ and reduces the values of Δp and $\Delta\epsilon + \Delta\lambda$ to $+''\cdot 014$ and $''\cdot 013$ respectively. The observations are compared with the curve $+''\cdot 0020 \cos l + ''\cdot 0060 \sin l$ in fig. 2.

Inspection of the diagram leaves little doubt of the existence of this term. The minimum may be too low owing to systematic errors in the right ascensions of stars between $-30°$ and $-60°$ Dec. It seems to me that $\Delta\epsilon$ should be found by calculation and $\Delta\lambda$ from discussion of the observations of the Sun in spite of their liability to systematic error, as the loss of weight, when this is combined with determination of precession, is very great.

Although these corrections to the precession and the motion of the equinox required by Newcomb's system are large, they nearly cancel in the motion in longitude, giving $+''\cdot 0008$. This leaves

$$-''\cdot 0046 \cos b + ''\cdot 0025 \cos 2(l-327°) \cos b$$

as a term in the longitude proper motions due to the rotation of the galactic system, when the systematic corrections of Raymond or Boss and Jenkins are applied.

I am indebted to Mr. Cullen and Mr. Symms for much arithmetical help, and for their assistance in the tabulation of latitude and longitude and calculation of the proper motions in these directions.